A Theoretical Model of Industrial Economy Inflationary Dynamics
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Abstract

One of the ongoing policy issues confronting monetary authorities around the world is the management of a stable price environment. Unstable prices create uncertainty, lower investment, and raise costs of doing business, thus lowering rates of growth. As a result, there exists a widespread need for understanding inflationary dynamics in any country of interest. This paper develops a standard monetary inflation model and augments it to include import, labor, energy, and intermediate goods and materials cost of production factors in a theoretically plausible manner. Implications for implementing an empirical version of the model are also discussed with a view toward the various econometric difficulties that may surface in estimation.

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Introduction
Almost every economy in the world must deal with the impacts of inflation in one manner or another. In general, inducing a stable price environment is regarded as a key step to improving economic welfare by enabling any given economy to operate more efficiently. This argument applies equally well to advanced economies such as the United States (Motley, 1993) as well as developing nations (Zind, 1993). Although the magnitude of disinflationary goals set by monetary authorities may differ, short-run price stabilization is frequently a center piece of government policy efforts worldwide. In cases where inflation is fairly low, maintenance of a stable price environment generally becomes one of the primary goals of central bank policy efforts. In both situations, it is essential to understand the dynamics of price movements if these goals are to be attained.

This paper presents a theoretical modeling framework for the purpose of analyzing industrial economy inflationary dynamics. A standard monetary model is adapted to incorporate factor costs of production, labor and imported inputs, in a mathematically consistent manner. Subsequent material in the study offers a review of the literature, theoretical model, and implications for empirical implementation. Suggestions for future research are summarized in the conclusion.

Literature Review
A seminal paper in the research on inflationary dynamics was actually conducted for developing countries by Harberger (1963). That early
paper interestingly points out that analyzing nominal data in level form
could result in spurious correlations in equations estimated for highly
inflationary economies. To circumvent this problem, percentage rates of
change are utilized in a linear regression framework based on the quant-
ity theory of money. What became known as the "Harberger" frame-
work incorporates real income, current and lagged values of the money
supply, and the opportunity cost of holding cash balances. The success
of this initial effort conducted on Chilean data spurred a series of
replicated studies for other developing countries (for example, see
Vogel, 1974). Results generally confirm the overall usefulness of the
Harberger model.

Following numerous applied econometric studies utilizing this ap-
proach, it became apparent that exclusive reliance on domestic variables
alone often provides unsatisfactory results. Bomberger and Makinen
(1979) provide a thorough examination of the Harberger model using
quarterly data for Korea, Taiwan, and Vietnam. Rather than extend
the model in a new direction, extensive testing is conducted in order to
establish whether a suitable characterization of inflation is provided.
Encouragingly, the parameter estimates do not appear sensitive to the
time period selected. However, the elasticities with respect to money
and real income are not always unitary as hypothesized. Also, the co-
efficient signs for the cost of holding money variables are sometimes
negative.

Hanson (1985) extends the Harberger framework in a systematic fash-
ion to incorporate an important missing component, import costs. An
implicit cost function is utilized to derive an aggregate supply curve
which includes local prices of imported inputs. When the underlying pro-
duction function is homogeneous of degree one, inflation becomes a
weighted sum of money supply changes and import prices. This is important because it implies that the elasticity of inflation with respect to money growth is less than one. Empirical results in the Hanson article strongly support the inclusion of import prices in models of inflation.

Subsequent research on inflationary dynamics in the United States provided additional evidence in favor of the augmented Harberger-Hanson approach wherein the effect of import prices on inflation is considered. Koch, Rosensweig, and Witt (1988) and Fullerton, Hirth, and Smith (1991) both report positive linkages between the trade-weighted exchange value of the dollar and consumer prices. These empirical studies indicate a unidirectional channel of influence from the exchange rate to domestic prices exists in the United States economy. As will be discussed below, causality direction has important implications for both model form and estimation technique.

Given the usefulness of the Harberger-Hanson framework, a natural question to ask is whether it can be further extended to systematically incorporate other variables that also impact upon price movements. Examples include labor, energy, and both intermediate and raw material prices (similar arguments are also presented in a different context by Dornbusch and Fischer, 1993). The models developed below are directed to this end. They are primarily intended to help model inflationary processes in industrial economies where data constraints are generally not binding. In principle, the augmented Harberger-Hanson approach can also be applied to developing economies, but data availability may preclude full implementation of the complete modeling system (for discussion, see Fullerton and Araki, 1996, and Sawyer and Sprinkle, 1987).

In Japan, the relation between the exchange rate and domestic prices
has been analyzed via so called Pricing To the Market (PTM) framework associated with Japanese corporations. The PTM approach differentiates domestic product prices from their corresponding export prices. There seems to have been a popularly supported hypothesis that even when the exchange rate (yen per dollar) falls, producers will not automatically raise the export prices in order to preserve overseas market shares. Simultaneously domestic prices will also be managed to earn extra-profits to cover the loss incurred by the overseas PTM strategy. Several articles has been analyzed such corporate PTM behavior and reported the estimation results rather favorable to this hypothesis. For example, Chida (1993) derives the theoretical equation by minimization of the generalized McFadden cost function under the restrictions of constant returns to scale, linear homogeneity in price, and so on to analyze the effect of the exchange rate on both domestic prices and export prices. Yamada and Yamanaka (1993) adapted Hooper and Mann's (1989) framework to analyze the effect of the exchange rate on the export prices. Based on mark-up pricing theory, they derive their theoretical model, “pass-through equation”, via minimization of the translog cost function with the restrictions of symmetry, linear homogeneity in price, and constant returns to scale. As will be shown below, the strategy underlying the extended Harberger–Hanson model in this paper is more straightforward with no more restrictions than these studies and will provide another framework to analyze this problem.

Still more, with regards to the macro economic analysis of inflation in Japan, Yoshikawa (1996) derives a theoretical equation in which various determinants of inflation are incorporated, labor costs, energy and raw material prices, money supply and so on. Yoshikawa (1996) uses this equation only to analyze inflation theoretically. As described below,
these factors will be incorporated in the extended Harberger–Hanson model which can be estimated directly.

**Theoretical Model**

Harberger’s (1963) model is based on the traditional quantity theory of money equation:

1. \( MV = PQ \),

where \( M \) represents some measure of the money stock, \( V \) is the velocity of circulation, \( P \) is the price level, and \( Q \) is real output. Unlike simple textbook examples of the quantity theory, Harberger enhances the realism of the model by allowing velocity to vary instead of arbitrarily forcing it to be constant. Velocity is assumed to be a predictable function of other macroeconomic variables that reflect the cost of holding cash balances.

To utilize percentage changes, the variables can be transformed by natural logarithms and then first differenced. Introduction of a time subscript, and rearrangement of the terms, yields the basic Harberger equation:

2. \( DP_t = DM_t - DQ_t + DR_{t-1} \),

where the last term results from substituting for velocity and \( D \) represents a difference or backshift lag operator. In contrast to the original version of this model, Equation 2 utilizes a one period lagged change in a representative interest rate series to proxy for the implicit cost of holding money. This approach can only be applied to modeling inflation in countries where government banking system regulations have allowed savings and loan rates to vary in both nominal terms and real terms. Unadjusted interest rates from economies in which this is not the case would obviously not provide accurate estimates for the cost of holding
idle cash. In economies where financial markets have not been permitted to operate flexibly, the lagged rate of change in an inflation index can be used to define the last explanatory variable in Equation 2 (Fullerton and Araki, 1996). The latter approach is not adopted here, thus yielding a slightly different specification than that appearing in Harberger (1963).

Equation 2 implies that inflation will vary positively with the money supply and inversely with respect to real output. A statistically significant intercept term will enter the estimated equation if there is a discernible trend in the velocity of circulation. If only contemporaneous lags of \( M \) and \( Q \) enter in the equation, the parameters for both variables are hypothesized to be unitary. This can be tested empirically with the following specification:

\[
DP_t = a_0 + a_1 DM_t - a_2 DQ_t + a_3 DR_{t-1} + u_3,
\]

where \( a_1 \) and \( a_2 \) are hypothesized to be positive, and the absolute values of \( a_1 \) and \( a_2 \) should both be statistically indistinguishable from one. The last argument in the expression represents the disturbance term.

Hanson (1985) proposes an implicit cost function dual of an aggregate production function which is homogeneous of degree one. Derived output supply functions from this framework will be homogeneous of degree zero in input and output prices. Equation 4 expresses this relationship using logarithmic first differences:

\[
DQ_t = b_0 + b_1 DP_t - b_2 DPI_t + u_4,
\]

where \( PI \) represents imported input prices. When the relative prices of imported inputs increase, output is assumed to decline. The standard homogeneity assumptions for production and derived supply relations imply that \( b_1 - b_2 = 0 \).

The vector of input prices utilized in the derivation of Equation 4 can
be extended to include factor prices beyond those represented by imports of materials, equipment, and services. Perhaps the most obvious candidate to further improve the relevance of the framework is labor costs. Doing so yields the following theoretical expression:

5. \[ DQ_t = b_0 + b_1 DP_t - b_2 DPI_t - b_3 DW_t + u_5, \]

where \( W \) represents wage and labor costs. In this case, the standard microeconomic assumptions for production and derived supply functions imply that \( b_1 - b_2 - b_3 = 0 \).

Equation 5 can be substituted into Equation 3 to eliminate the output term from the expression to be estimated. This step is exceedingly useful for avoiding interpolation bias in empirical studies of monthly inflation for countries where national income and product accounts are published at quarterly and/or annual frequencies (for a discussion of interpolation bias, see Bomberger and Makenin, 1979). The resulting equation can be written as follows:

6. \[(1 + a_2 b_1) DP_t \]
\[ = a_0 - a_2 b_0 + a_1 DM_t + a_2 b_2 DPI_t + a_2 b_3 DW_t + a_3 DR_{t-1} + u_6. \]

Equation 6 can be further simplified prior to estimation. Dividing through by the left-hand side constant term and rearranging terms such that the price series remains as the dependent variable generates the following relation:

7. \[ DP_t = c_0 + c_1 DM_t + c_2 DPI_t + c_3 DW_t + c_4 DR_{t-1} + u_7, \]

which also has testable properties. Note that the coefficient on the monetary variable, \( c_1 \), is hypothesized to be significantly less than one. With the possible exception of the intercept, all of the regression parameters in Equation 7 are expected to be positive. During periods such as the 1960s and 1970s in which inflation is accelerating, \( c_0 \) is likely to be greater than zero (Clavijo, 1987). Disinflationary episodes such as those
observed in many economies during the 1980s and 1990s may have negative intercept terms associated with them.

As indicated in the literature review, this general approach has provided a useful framework for analyzing quarterly and annual inflation rates. But because the implied lag structure is fairly short, it may require additional modification prior to estimation. His possibility does not reflect any deficiencies in the theoretical model as such, but arises due to the fact that data are published at different frequencies. As a result, if the inflationary impact of a change in the money supply is felt over the course of one calendar year, the implied lag structure for a model estimated with monthly data would potentially range up to 12 months in length. Equation 8 takes into account this empirical issue which has confronted and confounded researchers for many years (see Laidler, 1993):

$$DP_t = c_0 + c_1 DM_{t-i} + c_2 DPI_{t-j} + c_3 DPW_{t-k} + c_4 DR_{t-1-m} + u_8,$$

where $i, j, k, m = 1, 2, \ldots, M$.

Further modification to the framework yields an even more realistic reduced form specification. Essentially, all that is required is to expand the input price vector used in Equation 5 prior to substituting for the $DQ_t$ variable in Equation 4. It is generally agreed, for example, that domestic energy price measures and intermediate industrial good and material prices play important roles in the inflationary process. The latter tend to rise very rapidly during periods in which industrial capacity utilization is relatively high (Emery and Chang, 1997, and Price and Dockery, 1991). Let $E$ stand for energy prices and $I$ represent intermediate industrial good and raw material prices. Solving the modified specification of the model and allowing for an arbitrary lag structure results in the following reduced form:
9. \[ DP_t = c_0 + c_1 DM_{t-t} + c_2 DPI_{t-j} + c_3 DPW_{t-k} + c_4 DE_{t-m} + c_5 DI_{t-n} + c_6 R_t \cdot 1 - s + u_t, \]

where \( i,j,k,m,n,s = 1,2,\ldots,M \). Note that \( E_t \) represents energy prices and \( I_t \) stands for intermediate industrial good and material prices, both in time period \( t \). As before, all of the estimated slope parameters are expected to be smaller than one and have a cumulative sum that is positive. The constant term may be positive or negative.

Equation 9 provides an attractive starting point for examining inflationary trends in any given economy. It is not, however, without potential problems for analyzing price movements. The principal concern with this theoretical construct arises from the fact that Equation 9 treats all of the regressors as exogenous or pre-determined. In doing so, it does not allow for the possibility of statistical feedback or endogeneity between the left-hand and right-hand side variables.

If a central bank yields to political pressures and engages in accommodative monetary policy in the face of inflation shocks, this assumption would be violated. Koch, Rosensweig, and Witt (1988), as well as Fullerton, Hirth, and Smith (1991), report evidence that this is not the case in the United States. It would not be surprising, however, if feedback were encountered in at least some instances since it has been only in recent years that monetary policy in countries such as Canada and New Zealand has been freed to focus exclusively on price stability (Dornbusch and Fischer, 1993). In Japan, Yoshikawa (1996) reported that the anti-inflationary stance of the central bank strengthened substantially in the aftermath of the first oil crisis in 1974. Ultimately, the issue can only be resolved empirically. Careful testing should be utilized prior to selecting the methodology that will be employed in parameter estimation for Equation 9. As described in the next section, it seems
important for the structural changes of the relation or the movement of relevant variables to be taken into account.

A second possible concern arises from utilizing first differenced, log-transformed time series data in the equation to be estimated. If the resulting series are stationary, the equation can be estimated without risk of obtaining spurious correlations in the results. As shown in many studies of hyperinflationary economies, however, higher order differencing may be required to induce stationarity during periods in which prices increase rapidly (see Engsted, 1993). Because most, if not all, industrial economies have not experienced any hyperinflationary episodes during the post-war period, regular first differencing should adequately remove nonstationary trends in the first moments of any variables selected for analysis. It is still important, however, to formally test the latter assumption when empirical versions of Equation 9 are analyzed. For example, Soejima (1995) reports several recent studies that illustrate the possibility that the log-transformed time series price data become I(2) processes, i.e. the first differenced series are not stationary, only the second differenced series become stationary.

**Empirical Analysis**

In order to examine whether the working series included in Equation 9 are stationary, a variety of unit root tests may be utilized. For rare cases in which data limitations are severe, applying unit root tests to relatively short time spans may be risky. The latter consideration is due to the fact that these tests typically have low power unless long-run data sets are employed (Hakkio and Rush, 1991). Monthly time series data for consumer prices and exchange rates generally date back to 1957 and do not pose any problems of this nature. For other series such as money
supply aggregates or wages, there may be little that can be done to circumvent this potential problem. In such cases, chi-square tests calculated for autocorrelation functions (Greene, 1993) may provide corroborative evidence to support the results obtained from the unit root tests.

As noted above, some recent work indicates that log-transformed time series price data follow I(2) processes. This potentially necessitates some unit root testing and cointegration analysis for Equation 9. Complicating matters, however, recent time series research has shown that standard unit root tests may be unreliable when structural changes are observed in the trends of the variables under consideration (Perron, 1989). The first differenced, log-transformed time series data in Equation 9 might benefit from careful testing that directly incorporates potential structural changes into the unit root statistical framework (Soejima, 1995).

As specified above, the model is explicitly built around a set of unidirectional causality relations from movements in the regressors to consumer prices. To examine whether the absence of simultaneity in the model is plausible, Granger causality tests may be calculated for the stationary components of the series of interest. If the resulting F-tests fail to reject the feedback hypothesis, then the reduced form model appearing in Equation 9 is inadequate. Potential alternative candidate approaches include structural model systems of equations (Fullerton and Araki, 1996) as well as vector autoregression equation systems (Hamilton, 1994).

Some careful tests should be utilized to examine this problem because structural change may pose difficulties. As noted in the previous section, Koch, Rosensweig, and Witt (1988), as well as Fullerton, Hirth, and Smith (1991), report the possibility that there are no feed-
back effects from inflationary shock to monetary policy in the United States. Other authors, for example, Bernanke and Blinder (1992) have shown that the Federal Fund Rate responds positively to inflationary shocks and negatively to unemployment shocks. Accordingly a three variable VAR model is utilized to account for this endogeneity in the system. Similarly in Japan, Yoshikawa (1996) argues that the central bank effectively controls the short term call rate. That paper estimates a policy response function for Japan's call rate within a four variable VAR model including the call rate, an industrial production index growth rate, the percentage change in the consumer price index, and the ratio of net exports to production. Evidence is reported that the anti-inflationary stance of the central bank has been strengthened since the first oil crisis in 1974.

The model developed above accounts for more potential sources of inflation than do previous versions of the Harberger-Hanson theoretical framework. Its scope still may not be wideranging enough to explain all systematic variations in aggregate price levels for particular economies. If this is the case, the shortcoming will be manifested in the form of serially correlated residuals. If it is not possible to identify and/or extend the model to resolve the nonrandom component of the residuals, then the autocorrelation problem should be corrected in order to minimize the possibility of spurious estimation results (Hamilton, 1994). Even when valid theoretical respecifications can be identified, data constraints may preclude expansion of the model to account for the omitted regressors. In such instances, Pagan's (1974) nonlinear ARMAX procedure offers an attractive correction technique because of its capacity to handle autoregressive, moving average, and mixed error generating processes.

Expansion of the input price vector underlying Equation 4 to include
more than just import prices raises the risk of multicollinearity. Such an eventuality would, of course, make it difficult to correctly attribute the relative contributions of each variable to the overall inflationary process. In addition to the traditional recommendations for dealing with this problem (Pindyck and Rubinfeld, 1991), it may be helpful to experiment with alternative estimators. One candidate methodology is offered by ridge regression (for discussion, see Vinod, 1978). Because ridge regression has the undesirable side-effect of introducing a small level of bias in the resulting parameter estimates, it is mandatory that extensive simulation experiments be conducted with any model designed for usage in either policy analytic and/or forecasting exercises.

The lag structure in Equation 9 is arbitrary by design. In estimation, however, specific lag lengths must be selected. Because this question is an empirical one, experimentation with the lag structure generally cannot be avoided in any modeling process involving dynamic specifications. Fortunately, several decision rules are available. A formal statistical approach is provided by likelihood ratio model specification tests (Hamilton, 1994). Software packages frequently include automated versions of the Akaike and Schwarz criteria for dependent and independent variable lag inclusion (for descriptions, see Amemiya, 1980, and Hsiao, 1979).

Given the exploratory characteristics of the model developed herein, a variety of simulation exercises will be mandatory in examining empirical versions of Equation 9. The goal of the simulation tests is to provide insights with respect to the policy analytic and extrapolation accuracy of the underlying theoretical model. Usage of out-of-sample data not employed in parameter estimation provides the only means by which the simulation experiments can satisfy the Klein (1984) and Christ (1993)
criteria for forecast and model evaluation.

Conclusion

An inflationary model is developed above that includes both monetary and cost of production factor effects in a theoretically plausible manner. Specification and simulation of the model are relatively easy to accomplish. Because the model does not pose stringent data requirements, it is likely to be applicable to virtually any industrial economy of interest. Given its dynamic specification, the model may be useful in cases where monetary officials continue to grapple with short-term price stabilization goals.

As developed above, the theoretical model underlying the reduced form presented in Equation 9 represents an extension of Harberger–Hanson framework to include domestically produced factor costs. Further expansion of the input price vector utilized in deriving Equation 5 and the subsequent expressions is easy to accomplish. In cases where this is necessary, doing so will result in the inclusion of additional right-hand side variables in Equation 9, but will not alter the basic form of the model. Extending the input price vector along these lines, however, will raise the likelihood of encountering the aforementioned simultaneity and multicollinearity problems.

No matter what version of the dynamic framework is selected, numerous econometric issues may arise in estimation. With the exception of higher order first moment nonstationarity, the model can be adapted to handle most departures from standard statistical assumptions. It may be useful to expand the scope of the model to include additional equations that allow for potential endogeneity between inflation, money, wages, and exchange rates. Even if the latter are not required for parameter
estimation consistency, they could enrich subsequent policy simulation analyses. The model itself can be imbedded in structural model systems to accomplish this goal. Empirical research utilizing this modeling approach may help to unravel many of the questions remaining with respect to price trends in economies facing monetary policy decisions.

References


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