ZENO’S SECOND PARADOX
AGAINST MOTION,
THE ACHILLES

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I  PREFACE

Aristotle in the Physics Z 9, 239b15-18 reports Zeno’s second argument against motion as follows:

`δεύτερος δὲ ὁ καλοῦμενος Ἀχιλλεύς. ἔστι δ’ οὕτως ὅτι τὸ βλαδύτατον ὁδέποτε καταληψθῆσαι θέλειν ὑπὸ τοῦ ταχύτατος ἐμπρόσθεν γὰρ ἀναγκαῖον ἔλθειν τὸ διώκον ὃθεν ὅρμησε τὸ φεύγον, ὅστ’ ἀεὶ προέχειν ἀναγκαῖον τὸ βραδύτατον.’

(The second is the so-called “Achilles.” This is that the slowest runner will never be overtaken in its course by the swiftest, since the pursuer must first reach the point from which the pursued started, and so the slowest must always be ahead.)

It appears that the impression of paradoxical-ness of this second argument against motion comes substantially from the following circumstances: ‘The slowest (runner) will never be overtaken …by the swiftest,’ whereas you may consider that the Achilles is not so paradoxical as the Dichotomy which appeared to violate straightforwardly the principle of contradiction. Indeed, it appears that the text of the Dichotomy entails that ‘a thing in motion (τὸ φερόμενον) is not in motion (μὴ κινεῖσθαι).’ The
Dichotomy, so it seems, exhibits a plain contradiction. However, in spite of your objection, the purport of Achilles too, is in reality as paradoxical as Chuang-tzu the Taoist philosopher’s following words are paradoxical:

‘Nothing in the world is bigger than the tip of an autumn hair and Mount T’ai is small.’ 3)

By the way, Aristotle himself calls Zeno’s second argument against motion simply “the so-called Achilles” and treats it as essentially the same as the Dichotomy:

‘ἐστι δὲ καὶ οὕτως ὁ αὐτὸς τῷ διχοτομεῖν, διαφέρει δὲ ἐν τῷ διαίρεσθαι τό προσλαμβάνομεν μέγεθος, τό μὲν οὖν μὴ καταλαμβάνεσθαι τό βραδύτερον συμβεβήκεν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταύτῳ τῇ διχοτομίᾳ—ἐν ἀμφοτέρους γὰρ συμβαινεῖ μὴ ἀφικνεῖσθαι πρὸς τὸ πέρας διαριθμένου πως τοῦ μεγέθους· ἀλλὰ προσκείται ἐν τούτῳ ὅτι οὐδὲ τό τάχιστον τετραγωδημένον ἐν τῷ διώκειν τό βραδύτατον—ὁστ’ ἀνάγκη καὶ τήν λύσιν εἶναι τήν αὐτήν.’

(‘This argument is essentially the same as that depending on dichotomy, but differs in that the successively given lengths are not divided into halves. The conclusion of the argument is that the slower runner 4) is not overtaken, but it proceeds on the same lines as the dichotomy argument, since in both, being divided the distance in a given way, it is concluded that the goal is not reached: only in the Achilles a dramatic effect is produced by saying that not even the swiftest will be successful in its pursuit of the slowest and so the solution of it must be the same.’ 5)

It is a notable fact that Aristotle in the above quotation refers to the “dramatic effect” (τετραγωδημένον) of Zeno’s second argument. But, the saying seems to be somewhat beside the point, if we could not know Achilles’ rival. Who is Achilles’ rival?

According to Simplicius the commentator, Achilles’ rival is a tortoise. Simplicius insists that Zeno’s second paradox against motion, the
Achilles, was so called ‘because of the introduction into it of Achilles, who cannot possibly overtake the tortoise he is pursuing.’\(^6\) But, why does a tortoise make its appearance all of a sudden? It is, according to H. D. P. Lee, because of its dramatic and sensational effect.

Referring to the word-meanings of ‘τετραγωδημένον’\(^7\) Lee said as follows: ‘the introduction of Achilles and the tortoise in this [second argument], of the flying arrow in the next argument, and of the stadium in the last give them a certain vivid and dramatic quality, of which Aristotle evidently thoroughly disapproved, but which seems to have been very characteristic of Zeno’s reasoning.’ [italic by Yamakawa]\(^8\)

But, what does Aristotle’s disapproval in this context exactly mean? On this matter W. D. Ross said as follows:

‘In fact, practically the only detail that reaches us independently of him [Aristotle] is that Achilles’ rival in Zeno’s parable was a tortoise. This comes to us from the Greek commentators on Aristotle. The reference to the tortoise comes also from Plutarch, but in Plutarch the tortoise is matched not against Achilles but against “the fast horse of Adrastus,”\(^9\) so that possibly Achilles’ rival was not a tortoise at all, and two independent stories may have got mixed up in the familiar title.’\(^10\)

I do not know, of course, what kind of the opponent party of Achilles Ross had in mind by his saying that ‘possibly Achilles’ rival was not a tortoise at all.’ For my part, however, contrary to Ross’ conjecture, I surmise that Zeno himself regarded originally a tortoise as Achilles’ opponent party.

II

It is a definite fact that “the swiftest runner” (τὸ τάχιστον) refers to Achilles the hero. Then, why is “the slowest runner” (τὸ βλαστατον) a tortoise?
(1) In Homer’s *Iliad* we can witness an imposing figure of Achilles, who is pursuing at full speed Hector as though he were ‘a falcon (κύρκος) … the swiftest of winged things (ἐλαφρότατος πετεινών).’

(2) Homer in the above context likens Hector to “a trembling dove” (τρίφων πέλεια). Therefore, a pair of ‘Achilles: Hector’ in this context is likened to the pair of “a falcon: a trembling dove.”

(3) But, the supposition that “the slowest runner” in Zeno’s context is “Hector” is out of question. Hector is never a slowest runner. At *Iliad*, XXII. 164-166 a footrace between two heroes is depicted as follows: ‘These two heroes circled thrice with swift feet (καρπαλίμοισι πόδεσσι) about the city of Priam.’ Hector is not a tardy runner, but Achilles’ capable rival.

Who is, then, the slowest runner? According to Simplicius it was a tortoise (χελώνη). But, why is a tortoise Achilles’ rival? Why cannot Achilles’ opponent party be a trembling “dove” or a “fawn”? Is it not by any chance a forgery by Simplicius?

The last supposition however comes into collision with evidences. In the first place, Simplicius could presumably consult with a copy of Zeno’s original book or at least he could resort to ancient reports of Zenonian arguments from a good source such as one of Theophrastos. Second, there is a fact that, before and apart from Simplicius, Themistius too refers to the “tortoise” as Achilles’ competitor. In addition to this, third, Simplicius himself refers to Hector who is never overtaken by Achilles.

‘τῷ δὲ ἐπὶ ἀπειρὸν ἔλαττον ἄλλο ἄλλου διάστημα λαμβάνειν διὰ τὴν ἐπ’ ἀπειρὸν τῶν μεγετῶν τομήν, οὐ μόνον Ἦκτωρ ὑπὸ τοῦ Ἀχιλλέως οὐ καταλήψησε, ἄλλ’ οὐδὲ ἡ χελώνη.’

(‘And so by taking distances decreasing in a given proportion ad infinitum because of the infinite divisibility of magnitudes, we arrive at the conclusion that not only will Hector never be overtaken by Achilles, but not even the tortoise.)

In this context of Simplicius’ utterance, it is obvious that Simplicius has
in mind two parties of the running match; that is, (a) a party of Achilles and Hector and (b) another party of Achilles and a tortoise.

(a) Achilles  Hector
(b) Achilles  Tortoise

The diagram (a) is a familiar one for the reader of Homer’s *Iliad.* But, (b) is unfamiliar and unexpected one, which has, as Aristotle said, a dramatic effect and might caused a sensation among people. Then, who is the inventor of the diagram (b)?

It is a noteworthy fact that the above pair of (a) and (b) is a compressed expression derived from the following diagram:

Achilles  Hector
  Tortoise

And this diagram seems to hide its behind the third party whose one competitor is tortoise, but another competitor is unknown (x):

Achilles  Hector
  Tortoise

(x)

Someone has substituted “x” for “Achilles.” Then, who is “x” and who was the author of a pair “Achilles and Tortoise”? I surmise that “x” is a “hare” and the original author of “Achilles and Tortoise“ is Zeno of Elea.

III

Before the problem in question is settled, we must take up another problem, which invites us to adopt one of the alternatives; either (1) Zeno produced an argument against motion, the title of which was originally “Achilles and Tortoise;” or (2) Zeno did not give a particular title to the second argument, whereas in his context of argument he assigned a part
of “the slowest runner” to a tortoise and “the swiftest” to Achilles.

Reviewing the opening scene of Plato’s *Parmenides*, we find that the latter alternative appears to hit the mark. When the reading of a part of Zeno’s book finished, Socrates at 127d-e asked to hear the “hypothesis” of “the first argument” again. It is a noteworthy fact that Socrates does not make mention of their titles. It seems, therefore, they had originally no titles. On the other hand, the style of hypothetical argument presented by Zeno in the same dialogue is indeed reminiscent of Zeno’s genuine arguments against “plurality” preserved by Simplicius.\(^{18}\) They are very abstract and dialectical. But the matter does not always eliminate the case that Zeno appealed to people’s imagination.\(^{19}\) Gorgias’ ‘On What Is Not or Concerning Nature’ presents the parallel case.\(^{20}\) In the midst of developing very difficult and abstract argument Gorgias could effectively use some vivid words such as “a man flying,” “chariot running over the sea,” etc.\(^{21}\)

Now let’s return to the above mentioned question: ‘Why did Zeno adopt a “tortoise” as the competitor of Achilles? To this question I have no other hypothesis than that Zeno had presumably in mind Aesop’s fable “Tortoise and Hare” or one of its variations.

But, did Zeno really know this fable? Yes, I think so. The fable in question dates back at least to Zeno’s contemporary age. For it is included in the *Augustana Recension* whose primary source is possibly the *Aesopita*, the first collection of Aesopic fables published by Demetrius of Phalerum in the end of the fourth century B.C. or in the beginning of the third century B.C.\(^{22}\) Demetrius of Phalerum compiled the Aesopic fables which were in circulation for a long time among Greeks and which was published ‘as a handbook of materials intended primarily for the use of writers and speakers.’\(^{23}\)

It is a well-known fact that Aristophanes utilized often Aesop’s fables in his dramatization.\(^{24}\) Socrates in prison also was concerned with versification of Aesop’s fables in his last days.\(^{25}\) Similarly for Hesiod, Sophocles, Herodotus, Xenophon, Plato, and Aristotle too the animals like eagle, beetle, fox, lark, hare, and tortoise etc. making their
appearances in the Aesopic fables were, in greater or lesser degree, familiar ones. Thus, there is nothing to be wondered about a fact that Zeno himself also was familiar with Aesopic fables.

However, you may still ask me stubbornly: ‘Is it sure that Zeno was familiar with the fable “Tortoise and Hare”? I do not know exactly whether it was the case or not. But, it is sure that Menedemus the contemporary rhetorician of Plato was well informed of this fable. Once quoting the lines from the *Omphale*, a satiric drama of Achaus the tragicus, Menedemus attacked his political opponents:

‘Ere long the swift is overtaken by the feeble,
And the eagle (ἀετῶς) by the tortoise (χελώνη).”

Note that this is another version of “Tortoise and Hare.” The ‘eagle’ corresponds to a ‘falcon’ in the above-mentioned Homeric passage.

Thus, it is obvious that the Zenonian analogy of ‘The Swiftest : The Slowest = Achilles : Tortoise’

(Falcon) -- (Dove)
Achilles  Hector疤 Homer’s *Iliad*
(Swiftest) \\
Hare  Tortoise疤 Aesopic fable
(Slowest)

must have been very suggestive and impressive for everyone who has been already well versed in the Homeric analogy: ‘Achilles : Hector = Falcon : Dove’ as well as Aesopic fable ‘Tortoise and Hare’ or its variations.

As you know well now, to win the hearts of the people by arousing their vivid imagination was Zeno’s important strategy.
Now, if the above mentioned hits somehow the mark, I hope, the following imaginary dialogue between Achilles and Tortoise also may be regarded as a not so bad revision of Lewis Carroll’s witty tale. It is a sequel to the foottrace of Achilles and Tortoise which touches the core meaning of Zeno’s second argument against motion.27

Suppose the swift runner Achilles could overtake a tardy tortoise at a spot $n$, and now sitting triumphantly on the tortoise’s back, he arrogantly says as follows:

**Achilles:** ‘I have conquered you. Now, you must admit your loss!’

**Tortoise:** ‘Hmmm … but, I cannot believe my … rather unpleasant … state of affairs. Disclose your trick, Oh, no! …but strategy, or way of running, so I may concede my loss.’

**Achilles:** ‘Well, listen to my words carefully and satisfy yourself of your miserable situation. My way of running was a splendid one. In the first step, I covered a distance greater than a half of the distance which could be shown as a length between the starting point and your final spot here. And in the second step, I covered a distance greater than a half of that which was left. Thus repeatedly employing such a strategy, I finally overtook you, and now I am resting comfortably on your back. Do you understand?’

**Tortoise:** ‘Oh, I see. Your way of running was really based on the Axiom of Archimedes.’

**Achilles:** ‘Not ‘Archimedes’, but Euclid’s *Elements* X. Prop. 1. In any way you are not enlightened about the history of mathematics.’

**Tortoise:** ‘Don’t mind such a trifling matter. Well, I recommend you to listen to Zeno’s argument. He says: “Even the slowest runner, when in motion, can never be overtaken by the fastest, for, inevitably, the pursuer must first reach the point from which the pursued started; it means that the slowest runner will always remain ahead.”

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27
ZENO’S SECOND PARADOX AGAINST MOTION, THE *ACHILLES*

If your way of running were Zeno’s, I could never be overtaken by
you, since he assures us that the two distances covered by you and
me were really incommensurable.

The *Achilles* is, according to Aristotle, ‘essentially the same as that
depending on the *Dichotomy*, but differs in that the successively given
lengths (*τὸ προσλαμβανόμενον μέγεθος*) are not divided into halves.’ Therefore, it is beyond doubt that in this argument too the motion is viewed in
the aspect of the perfect tense and its subject is the problem of the
between-ness and the denseness of extension. But, the *Achilles* can be regarded also as a representation of the process of successive subtraction
between two incommensurable quantities. Here in this context, with
prominent free use of *reductio ad absurdum* and possibly of the theory of
“even and odd” which was employed in Euclid’s *Elements* VII and IX, the
incommensurability between two distances traversed by Tortoise and
Achilles becomes afresh to be actualized in connection with the method of
“successive subtraction” (*ἀνθυφαίρεσις* or *ἀντανάφαίρεσις*).

28
NOTES

1) Bekker reads βραδότερον. But I take βραδότατον following Cornford, Lee, and Ross.

2) 'πρῶτος μὲν ὃ περὶ τοῦ μὴ κινῆσθαι διὰ τὸ πρῶτον εἰς τὸ ημισυ δεὶ κατοίκησθαι τὸ
φερόμενον ἢ πρὸς τὸ τέλος, περὶ οὔ διείλομεν ἐν τοῖς πρῶτοις πρῶτον λόγοις. διό καὶ ὃ Ζήνως
λόγος ψεύδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἀπείρα διελθέν ἢ ἂνασθαί τῶν ἀπείρων καθ’
εκαστὸν ἐν πεπερασμένῳ χρόνῳ. διὰ γὰρ λέγεται καὶ τῷ μήκος καὶ ὁ χρόνος ἀπείρον,
καὶ ὅλος πᾶν τὸ συνεχῆς, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἰσχάτοις. τῶν μὲν οὖν κατὰ ποσὸν
ἀπείρων οὐκ ἐνδέχεται ἄφασθαι ἐν πεπερασμένῳ χρόνῳ. τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται
καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἀπείρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συ-
μβάνει διαίνει τὸ ἀπείρων καὶ ἁπτεσθαι τῶν ἀπείρων τοῖς ἀπείρως, οὐ τοῖς πεπερασμένοις.'
('The first says that an object in motion is not in motion, because a thing in motion
must reach the half-way point before it gets to the end. This we have discussed above. Hence Zeno’s argument makes a false assumption when it asserts
that it is impossible to traverse an infinite number of positions or to make an in-
finite number of contacts one by one in a finite time. For there are two senses in
which length and time and, generally, any continuum are called infinite, namely
either in respect of divisibility or of extension. So while it is impossible to make
an infinite number of contacts in a finite time where the infinite is a quantitative
infinite, yet it is possible where the infinite is an infinite in respect of division;
for the time itself is also infinite in this respect. And so we find that it is possible
to traverse an infinite number of positions in a time of in this sense infinite, not
finite; and to make an infinite number of contacts because its moments are in
this sense infinite, not finite.)

3) 「天下，莫大於秋豪之末，而大山為小」（莊子『齊物論』篇）; Cf. A. C. Graham,

4) Following Bekker’s reading βραδότερον at 239b21 Lee retains it, whereas he
emended other βραδότερος of Bekker’s text to βραδότατον. However, he mistranslated
this βραδότερον to ‘the slowest runner.’

5) The text of the Achilles (Second part): ἐστι δὲ καὶ οὕτως ὁ αὐτὸς τῷ διχοτομῆν, δια-
φέρει δὲ ἐν τῷ διαμένει μὴ δίχα τὸ προσλαμβανόμενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβά-
νεσθαι τὸ βραδότερον συμβέβηκαν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταύτῳ τῇ διχοτομῇ—ἐν
ZENO’S SECOND PARADOX AGAINST MOTION, THE ACHILLES

ἀφιερυσμένοι γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρας διαιρομένου πιος τοῦ μεγέθους·

ἀλλὰ προσκαίται ἐν τούτῳ ὅτι οὐδὲ τὸ τάχιστον τετραγωδημένον ἐν τῇ διώκειν τὸ βραδύτατον—ἀστεὶ ἄναγκη καὶ τὴν λόγιν εἶναι τὴν αὐτήν.

6) See Simplicius, 1013. 31; Lee, p. 51: καὶ οὕτως ὁ λόγος ἐκ τῆς ἐπί άπειρον διαιρέσεως ἐπικεχείρηται κατʼ ἄλλην διασκέδαιν. καὶ εἱ ἐν τοιούτοις· εἰ ἔστι κίνησις, τὸ βραδύτατον ὑπὸ τὸ ταχύστου οὐδέποτε καταληψθήσεται· ἀλλὰ μὴ τοῦτο ἀδύνατον· οὐκ ἄρα ἔστι κίνησις ... (1014.9) Ἀχιλλέως ὡς ὁ λόγος ἀπὸ τοῦ παραληφθέντος ἐν αὐτῷ Ἀχιλλέως ἰκλήθη, ὕπ’ ἀδύνατον φησιν ὁ λόγος τὴν κελώνην διώκοντα καταλαβέν.

7) Concerning the word-meaning “pompous”, “bombastic” of τραγωδεῖν, τραγῳδία, Lee prompts us to see LS (J).

8) H. D. P. Lee, Zeno of Elea, A Text, with Translation and Notes, Amsterdam, Adolf M. Hakkert-Publisher, 1967, p. 77.

9) Cf. The Iliad, XXIII. 346.


11) Cf. The Iliad, XXII. 139-140: ἢ τέ κύρκος δρέσφιν, ἔλαιφότατος πετεινὼν,

12) Ibid: ῥητίδος ὄμησε μετὰ τρέφοντα πέλειαν ... In the other context, however, Homer likens Achilles to “a hound” (κύων) and Hector to “a fawn” (νέβρος). Cf. The Iliad, XXII. 189-190.

13) Cf. The Iliad, XXII. 199-201: ‘And as in a dream a man availeth not to pursue one that fleeth before him—the one availeth not to flee, nor the other to pursue—even so Achilles availed not to overtake Hector in his fleetness, neither Hector to escape.’ (ἀς δ’ ἐν δείρῳ οὐ δύναται φεύγοντα διώκειν/οὕτ’ ἀρ’ ὁ τὸν δύναται ὑποφέυγειν οὐθ’ ὁ διώκειν/ οὐ δύνατο μάρψαι πόσιν, οὔτ’ δ’ ἄλυξαι.) Translation by A. T. Murray (Loeb).


16) Cf. Themistius, 199. 25.

17) Simplicius, 1013.31

18) Saying this I have in mind Zeno’s two fragments B2 and B3 on which I shall give a detailed explanation later.

19) R. E. Allen considers that the abstract style of Zeno’s argument does not fit the “vivid” imagery of the arguments preserved by Aristotle. See his Plato’s
Parmenides, p. 79.
20) In his treatise ‘On What Is Not or Concerning Nature’ Gorgias the sophist developed a discourse which may be regarded as a parody or a witty reductio of Eleatic metaphysics utilizing Eleatic logic at important points of which included some vivid images appealing to imagination: “a man flying,” “chariots running over the sea,” “Scylla,” and “Chimaera.” Cf. DK, B3, 79; 80.
23) Ibid.
24) Cf. Aesopica 3; 26; 111; 177; 423; 428; 438; 447.
25) Cf. Plato, Phaedo 60d.
28) For the detailed explanation of the matter, see my paper ‘The Mathematical Structure of Zeno’s Paradoxes of Motion,’ in the above cited book, pp. 76-86.
SUMMARY OF
ZENO’S SECOND PARADOX
AGAINST MOTION, THE *ACHILLES*

Hideya YAMAKAWA

This paper is a modified excerpt from my original paper ‘The First Layer of Zeno’s Paradoxes’, which investigates into the hidden dimensions of Zeno’s argument against motion. I hope that the reader should regard this as a twin brother of ‘『もしもしかめよ』考’ (A Study of the Song ‘Moshi-moshi Kameyo’) which will be published on this volume at the same time.