

ZENO'S FOUR PARADOXES AGAINST MOTION

Hideya YAMAKAWA*

«Quand je dis donc que tout visible est invisible, que la perception est imperception, que la conscience a un «*punctum caecum*», que voir c'est toujours voir plus qu'on ne voit, ... il faut comprendre que c'est la visibilité même qui comporte une non-visibilité.¹⁾»

Merleau-Ponty, *Notes de travail*, “Visible-Invisibl ,” Mai 1960

* 本学法学部

キーワード : Zeno's Paradoxes, Visible and Invisible, *Dichotomy*, *Achilles*, *Arrow*, *Stadium*

THE FOUR PROVISIONAL EPISODES²⁾

1 ACHILLES AND HIS SHADOW³⁾

Achilles rushed
Like a wind on the plain of Scamander,
Achilles dashed
A thousand fold more quickly than the sun
Which was moving westward
From high overhead.

Far away, in the distance,
He saw a small, yes very tiny appearance
Of Trojan stronghold and
Of the glittering top of a tower
Reflecting sublimely
The light of the burning sun.

Now, you must wreak revenge upon that fellow
Hector who killed your beloved Patroclus.
Hurry, now, Achilles
Show him your matchless power,
You “light-footed”
Achilles!

ZENO'S FOUR PARADOXES AGAINST MOTION

Achilles rushed
Like a whirlwind on the blue meadow.
Achilles dashed
With his back to the setting sun,
And in front of him
Treading on his black shadow.

And the Shadow guided you, Achilles!
Like an anonymous clown or Cartesian daemon
Dancing and jumping,
Stretching and shortening his figure
Amorphously
On the dark green meadow-canvas.

2 DEATH OF THE SHADOW⁴⁾

There was a man who was afraid of his own shadow and his footprints. He was a deathly pale person who covered his whole body with a black hood and black cloak. And he was himself a shadow-like person. He believed that he might not always be followed by his own shadow if he himself was a ghostly fellow.

However his shadow and his footprints have tenaciously stayed close to him, appearing from somewhere unexpected whenever he went out. And each time they appeared he ran for his life, screaming with fear, in order to separate himself from them. But, with weird stretching and shrivelling, the shadow always crawled and wiggled there on the earth, sometimes in front of him, sometimes behind him.

Hoping to outstrip and detach himself from the fellow, the man put his best foot foremost. But he could neither overtake nor run away from that fellow, however hard he might try to pursue or escape. The more he ran, the greater became the number of his footprints, and his fear grew all the more. Thus, the man ran even harder.

Running and covered with sweat, he stupidly thought “my running is not enough.” Thus the man continued his desperate running incessantly and finally he expired by the roadside. There on the earth remained only the black hood and cloak, and the faintest of shadows.

3 THE DIALOGUE BETWEEN THE SHADOW AND ITS SHADOW⁵⁾

The shadow of the shadow complained against the shadow,
“Hey, you fellow, there is no-one as inconstant as you. A little while ago you were standing, and now you are sitting. For me this is unendurable as I must always move as you move. What an unprincipled chap you are! Be consistent!”

Responding to this the shadow said,
“I am at a loss when hearing such an irrational complaint, since, you know, not only you, hey, but also I myself must move as my master moves. However the master himself is in the same situation because my master is somewhat, you know, a puppet of a puppeteer.”

ZENO'S FOUR PARADOXES AGAINST MOTION

4 IMMOVABLE SPEAR⁶⁾

One pursuing and the other being pursued three times
Achilles and Hector ran about the castle wall of Ilium. And
At last for the fourth time when they came near to the fountain,
The two heroes confronted each other.

With his back to the sun Achilles
Threw his spear at Hector who protected himself with a buckler.
And the bronze spear which departed from Achilles' hand
Flew straight and fast.

In reality however it did not fly in the slightest.
In the space between the two heroes
Casting its black straight shadow on the burning field of Ilium
In midair it stood immovable.

II ZENO'S FOUR PARADOXES

«Le visible est pregnant de l'invisible.⁷⁾»

Merleau-Ponty, *Notes de travail*, Novembre 1959

1 ZENO OF ELEA

Once in the milieu of the ancient *Magna Graecia*, there lived a Greek called Zeno. He was a native of Elea, the Phocaean colony founded in the sixth century BC,⁸⁾ situated on the south-west coast of Italian peninsula.⁹⁾

Zeno was a disciple of Parmenides the founder of Eleatic school¹⁰⁾ and he himself was a renowned philosopher; his nickname was "Eleatic Palamedes."¹¹⁾

Though it appears that he had a few opportunity to visit Athens and give lectures to the public figures like Pericles and Callias son of Calliades,¹²⁾ yet he was primarily an enthusiastic patriot and so ‘despised the great no less than Heraclitus’ that he preferred to spend a large part of his life in his native Elea to staying at Athens.¹³⁾

Aristotle called Zeno “the inventor of dialectic.”¹⁴⁾ He was a philosopher especially gifted with dialectical reasoning and wrote possibly a book including the famous four arguments against motion.¹⁵⁾ But he was not only a philosopher but also a politician.¹⁶⁾ Like almost all Presocratics, he too committed to the political affairs. A number of stories inform us of his rebellion against a tyrant and of his heroic end under the terrible torture.¹⁷⁾

By the way, the arguments produced by him too, it appears, have the notes of such a rebellious or a retaliatory spirit. Plato in the *Parmenides* makes Zeno confess the true purpose of his book as follows:

The book is in fact a sort of defence of Parmenides’ argument against those who try to make fun of it by showing that his supposition, that there is a One, leads to many absurdities and contradictions. This book, then, is a retort against those who assert a plurality. It pays them back in their own coin with something to spare, and aims at showing that, on a through examination, their own supposition that there is a plurality leads even more absurd consequences than the hypothesis of the One.¹⁸⁾

The setting of the dialogue *Parmenides*, which assumes the meeting in Athens of three famous persons, Parmenides, Zeno, and young Socrates, may be a fiction. But, as almost all Plato scholars agree, we have no positive reason

ZENO'S FOUR PARADOXES AGAINST MOTION

to doubt what Plato says about the contents and purpose of Zeno's book.

2 ZENO'S PARADOXES

Zeno retorted against persons who ridiculed his teacher Parmenides. His argument was so sharp and powerful that it was difficult to resist his power of persuasion. Yet the matter did not affect that his conclusion was very perplexing and strange. In particular his argument against motion appeared to be quite illogical and senseless. He declared outrageously: "*An object in motion is not in motion.*"

Being confronted with such an apparently absurd conclusion from commonsensical viewpoint,¹⁹⁾ everyone raised loudly their objections all in one voice: "Why?", "Warum?", "Pourquoi?" Thus for a long period from the fifth century B.C. to our contemporary time, the noisy boing and disputations against Zeno's arguments have broken out and indeed lasted to be so until now our present day. Indeed, the "Zeno's paradox" became a pronoun of a long-standing and unsolvable problem. Thus each and every sort of heads were, irrespective of the age or gender, irrespective of their shape and colour, and of course irrespective of their hair-style and a quantity of hair, lost in their thought and inflamed with the notorious "paradoxes," which Zeno of Elea skilfully produced 2,500 years ago.

Everyone knows somehow of Zeno's paradox. You must possibly remember the "Achilles and Tortoise:" It declares that even Achilles the hero and the swiftest runner in the Trojan War, can by no means overtake the tortoise which goes on all fours ahead very slowly. Some of you are surely to be reminded of your former classroom scene when you were still boys and girls; or of your teacher's or of your friend's feverish chattering about the paradox; of

an irritating riddle under your nose. Then, with your tongue-clicking, you might have given a vexatious groan: “Devil takes it!” And at last becoming very serious you might have scribbled on your desk some clumsy illustrations of the two competing figures of Achilles and tortoise as well as many jumbled lines or calculation numbers with your stubby pencil!

So famous is Zeno’s paradox, though of course there is no testimony about a fact that Zeno himself or his contemporaries called Zeno’s argument against motion “paradox.” It was indeed in later ages that Zeno’s discourses in the lump came to be called “paradoxes.” The fact discloses implicitly the posterity’s way of evaluation of Zeno’s personality as well as of his discourse.

The “paradox” in English is derived from a Greek adjective “παράδοξος” (*paradoxos*) which was originally compounded from a Greek proposition “παρά” (*para*) (the pronoun in this case means “contrary to,” “anti” etc.) and a noun “δόξα” (*doxa*) (the noun in this case means “opinion,” “judgement” etc.) and customarily used among the public to mean for example the cases such as “a statement which seems to be foolish, anti-commonsensical or impossible,” “an improbable combination of the opposing qualities, ideas, etc.,” “a statement, idea, or fact that is combination of the opposing qualities, ideas, etc.,” and “a statement, idea, or fact that is opposite or contrary to what is generally or commonsensically believed to be true or to be just.”²⁰⁾

The “paradox” is generally in our everyday life regarded as a kind of “fun” at best. But it appears that the “paradox” bears something more serious than “fun.” It puts somehow on itself a more critical or negative nuance; in other words it has a slanting posture and an indication of the immediate counterattack against a serious treatment, which can in its extreme case torment one so as to cause his death.²¹⁾ So, it may be considered that a “paradox” is nothing

ZENO'S FOUR PARADOXES AGAINST MOTION

but a false charge to a just thesis; that is to say, it is analogous to a negative picture against a positive one, i.e., a mere shadow against the original. Thus, they might say that if there is no established doctrine there is no room for a paradox too; therefore, a paradox remains to be a negative proposition in the long run, whose fallacious roots should be rightly disclosed by a commonsensical deliberation.

Thus they concluded that there is something wrong in the “paradox;” in other words some non-obvious flaws or some camouflaged facts, since a “paradox” eventually means that ‘an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises.’²²⁾

3 ZENO AS A SOPHIST

So people thought about the case of Zeno's paradoxes too. They regarded Zeno's paradoxes as just skilfully constructed tricks or a kind of sophisms at the most. In short, people have scornfully regarded Zeno himself as a sophist, whose identity was a mere trickster of words and a plain “double-tongued person (ἀμφοτερογλώσσος).”²³⁾ However, it is a noteworthy fact that the judgement by B. Russell was different from this. Deploring the lack of critical eye of the posterity, he said in his *Principles of Mathematics* as follows:

In this capricious world, nothing is more capricious than posthumous fame. One of the most notable victims of posterity's lack of judgement is the Eleatic Zeno. Having invented four arguments, all immeasurably subtle and profound, the grossness of the subsequent philosophers pronounced him to be a mere ingenious juggler, and his arguments to be one and all sophisms.

According to Russell's judgement, Zeno of Elea was not a mere "juggler" but a genuine philosopher who pioneered a new and bold approach for the future research into the notoriously difficult problems of "infinity" and "continuity."

Setting aside at the present whether or not Russell's judgement is proper, in this volume I want to take up Zeno's four arguments in question, i.e., the *Dichotomy*, the *Achilles*, the *Arrow*, and the *Stadium* from another point of view afresh and show a fundamental significance of Zeno's paradox.

4 VISIBLE AND INVISIBLE

Notifying beforehand my intention in this volume, I would like to convey here some of my true motive. I have no intention to deal with Zeno's discourse qua "puzzle." And much less I intend to issue some new and unexplored "solutions" of Zeno's discourses qua "puzzles," which might contrive the intricate mathematical devices and which could be appreciated only by a small circle of specialists. My original intention in this volume lies in showing a fact that something "invisible" is hidden behind the "visible" surface of Zeno's discourses. Zeno's "paradoxes" as "visible" strata are in reality pregnant of the "invisible" meaning, by which Zeno's real intention can be disclosed. The meaning is "invisible," but the "invisible" meaning in question is not separated from its "visible" layer. The "visible" thing itself assumes the meaning of the "invisible." Thus, the "invisible" is a secret counterpart of the "visible."²⁴⁾

I said just a moment ago that I have no intention to deal with Zeno's discourses qua puzzles. Yet, the subject-matter he called into question and indeed Zeno himself hide behind the "visible" surface qua "puzzle." Therefore, in order to excavate Zeno's "invisible" figure and bring it to light successfully, we

ZENO'S FOUR PARADOXES AGAINST MOTION

must remove a large quantity of surface soil which covers Zeno's true identity. Thus breaking through the bulky layers of "paradoxes," we have, by all means, to get to the solid rock of Zeno's thought.

I shall begin my task immediately and after digging up a great deal of earth, I hope I could show you a fact that Eleatic philosopher's keen insight, indeed, penetrated fully into a rock bed of human intelligence.

5 ARISTOTLE'S REPORTS ON ZENO'S PARADOXES

It is regrettable however that there is none of Zeno's own genuine four discourses on motion left, which Russell praised as being "all immeasurably subtle and profound." It is true that there are some other genuine fragments of Zeno. But, they are scanty and meagre. To investigate the four arguments in question, we have no other means of relying on Aristotle's valuable but indirect information, especially on his report of Zeno's arguments in the *Physics* Z 9, 239b5-240a18. Besides, there is another disappointing fact. Aristotle was not always so friendly to Zeno. It appears that he had a rather bit hostile feeling against Zeno. Whenever he refers to Zeno's arguments, as I see it, he had an underlying motive to bring forth a counterargument to Zeno. Therefore, we might not place so much our hope on Aristotle's unbiased view of Zeno's discourses. In spite of this discouraging circumstance, however, we must always keep a stark fact that without relying on Aristotle's report, there is no way to grope for the hidden dimension of Zeno's thought. So, in the first place, at any cost, we must check closely Aristotle's words in *Phys.* Z 9 one by one.

Aristotle at *Physics* Z 9, 239b9 begins his report of Zeno's discourses with the following impressive words:

‘There are four arguments about motion of Zeno which give trouble to those who try to solve the problem they involve.’²⁵⁾

And he goes on to enumerate Zeno’s four arguments against motion one after another. His report is so extremely compressed that the explanation is hard all the more for its elliptical method of expression.

Let us leave a detailed study of Aristotle’s argument. Here in this introductory Chapter it is sufficed to pass an eye over the text and grasp the main point in which each argument’s paradoxical-ness consist. In the following, I quote all the text of four arguments in translation, distributing Greek texts themselves for the critical reference into the column of footnotes. In giving a headline to each argument, for the convenience, following the established practice I use the term “paradox.” Thus, I call Zeno’s first argument against motion “The first paradox” and so on.

6 THE FIRST PARADOX: THE *DICHOTOMY* (239b11–13)

‘The first says that an object in motion is not in motion, because a thing in motion must reach the half-way point before it gets to the end. This we have discussed above.’²⁶⁾ Hence Zeno’s argument makes a false assumption when it asserts that it is impossible to traverse an infinite number of positions or to make an infinite number of contacts one by one in a finite time. For there are two senses in which length and time and, generally, any continuum are called infinite, namely either in respect of divisibility or of extension. So while it is impossible to make an infinite number of contacts in a finite time where the infinite is a quantitative infinite, yet it is possible where the infinite is an infinite in respect of division; for the

ZENO'S FOUR PARADOXES AGAINST MOTION

time itself is also infinite in this respect. And so we find that it is possible to traverse an infinite number of positions in a time of in this sense infinite, not finite; and to make an infinite number of contacts because its moments are in this sense infinite, not finite.²⁷⁾

Pay attention to a fact that the text runs as follows: ‘A thing in motion (τὸ φερόμενον) is not in motion (μὴ κινεῖσθαι), because (διὰ) ...’ The sentence structure itself exhibits a plain contradiction. In order to derive this contradiction the subordinate clause beginning with ‘διὰ’ (because) is introduced. Postponing a full interpretation of its implication, for the present, it is opportune to imagine a runner who is on the track in a stadium, since Aristotle at the *Topics* 160b8–9 refers to this argument of Zeno as one purporting the impossibility of traversing the “stadium.”²⁸⁾ The case suggests us an important fact. Zeno’s *Dichotomy* was originally an argument which started from making an appeal to our imagination.

7 THE SECOND PARADOX: THE *ACHILLES* (239b15–18)

‘The second is the so-called “*Achilles*.” This is that the slowest runner²⁹⁾ will never be overtaken in its course by the swiftest, since the pursuer must first reach the point from which the pursued started, and so the slowest must always be ahead.’³⁰⁾

The paradoxical-ness of this argument consists in the following: ‘The slowest (runner) will never be overtaken ...by the swiftest.’ You may consider the *Achilles* is not so paradoxical as the *Dichotomy*, which appeared to straightforwardly violate the principle of contradiction. In reality however it is as

paradoxical as Chuang-tzu the Taoist philosopher's following words are paradoxical: 'Nothing in the world is bigger than the tip of an autumn hair and Mount T'ai is small.'³¹⁾

Aristotle calls this argument simply "the so-called *Achilles*" and treats it as essentially the same as the *Dichotomy*:

This argument is essentially the same as that depending on dichotomy, but differs in that the successively given lengths are not divided into halves. The conclusion of the argument is that the slower runner³²⁾ is not overtaken, but it proceeds on the same lines as the dichotomy argument, since in both, being divided the distance in a given way, it is concluded that the goal is not reached: only in the *Achilles* a dramatic effect is produced by saying that not even the swiftest will be successful in its pursuit of the slowest and so the solution of it must be the same.³³⁾

Aristotle refers to the "dramatic effect (τετραγωδημένον)" of this argument. But, it does not especially so appear, because we cannot detect a concerned party of the race.

On this topic I will try to give an explanation later.

8 THE THIRD PARADOX: THE ARROW I (239b5-7)

'Zeno argues fallaciously. For if, he says, everything is either at rest or in motion, but nothing is in motion when it occupies a place equal to itself, and what is in motion is always in a given now occupies a place equal to itself, then the arrow in motion is not in motion. But this is false; since time is not composed of a number of indivisible now any more than any

ZENO'S FOUR PARADOXES AGAINST MOTION

other magnitude is composed of indivisibles.³⁴⁾

9 THE THIRD PARADOX: THE *ARROW* II (239b30–33)

‘The third is that just given above, that the arrow in motion is at rest. This conclusion follows from the assumption that time is composed of a number of now; for if this is not granted the conclusion cannot be inferred.’³⁵⁾

Zeno argues that a flying arrow does not fly. This is also a vivid and really astounding stunt. By the way, Aristotle refers to this twice. This is the reason why I distinguished the *Arrow* I from the *Arrow* II.

The *Arrow* I is brought forward by Aristotle in the opening of the chapter 9 with a sharp toned utterance that ‘Zeno argues fallaciously’ (Ζήνων δὲ παραλογίζεται),³⁶⁾ whereas the *Arrow* II is assigned to the third position (“the third” (τρίτος)) in relation to the other arguments against motion.

In both cases, however, Aristotle calls Zeno’s use of “now” (τὸ νῦν) into question; in the *Arrow* I “a number of ‘indivisible’ now,” and in the *Arrow* II “a number of now” without a definitive “indivisible.” Namely, Aristotle in *Arrow* I insists: ‘This is false; since time is not composed of a number of indivisible’ (τοῦτο δ’ ἔσται ψεῦδος· οὐ γὰρ συγκείμεται ὁ χρόνος ἐκ τῶν νῦν τῶν ἀδιαίρετων) and in *Arrow* II: ‘This conclusion follows from the assumption that time is composed of a number of now.’ (Συμβαίνει δὲ παρὰ τὸ λαμβάνειν τὸν χρόνον συγκείσθαι ἐκ τῶν νῦν). In any case Aristotle here sees the cardinal point at issue in the concept of “now” (τὸ νῦν).

Zeno in the third paradox says: ‘The arrow which is in motion is not in motion.’ This is also a plain contradiction. The absurdity of the *Arrow*’s

conclusion is in a close match with one of the *Dichotomy*'s.

10 THE FOURTH PARADOX: THE *STADIUM* (239b33–240a18)

‘The fourth is the argument about the equal bodies moving from opposite directions alongside the equal bodies in the stadium—the ones from the end of the stadium, the others from the middle—with equal speeds. In this setting, he thinks, it follows that half the time is equal to its double. The fallacy lies in the assumption that a body moving at an equal speed takes equal time in passing a body in motion and a body of the same size at rest. This is false. For example, let the bodies of equal size which are stationary be *AA*; let *BB* be those starting from the middle point of the *As*,³⁷⁾ being equal in number and in magnitude to *As*; and let *CC* be those starting from the goal, being equal in number and magnitude to *As*, and moving at equal speed with the *Bs*. Then it follows that the first *B* and the first *C* are at the end at the same time, as they are moving past one another. And it follows that the ⟨first⟩ *C* has passed all the *Bs*³⁸⁾ whereas the ⟨first⟩ *B*³⁹⁾ half; so that the time is half, for each takes an equal time in passing each body. And it follows that at the same time the first *B* has passed all the *Cs*. For the first *B* and the first *C* will be simultaneously at opposite ends of *As*, [being an equal time alongside each of the *Bs* as alongside each of the *As*, as he says,]⁴⁰⁾ since both take an equal time passing the *As*. That is the argument, and it rests on the above-mentioned falsity.⁴¹⁾

As you see, Aristotle’s report of this paradox is very lengthy and intricate; it is not only intricate but also obscure; in addition, the readings of the

ZENO'S FOUR PARADOXES AGAINST MOTION

transmitted Greek texts themselves are so manifold that the interpretations too are rather diverse. Here in this introductory part of my explanation, except for the following illustration of the *Stadium* it is suffice to tentatively follow the traditional reading of Lee in the *Zeno of Elea* in which he consulted to Simplicius' authoritative interpretation.

The scene again is set in a "stadium." Zeno in this argument also seems to be at pains to give a vivid representation. Alongside the one stationary even-numbered⁴²⁾ bodies A_1A_2 whose central axis (M) exactly corresponds to the middle-point of the stadium, the other two sets of bodies (B_1B_2 and C_1C_2) equal in number and size to the stationary one (A_1A_2) begin to move with equal speeds towards the opposite direction at the same time.

In the following figure 1 let S be the starting line and G be the goal in the stadium; and let it be that the A_1A_2 is a train of bodies which consists of two parts A_1 and A_2 in equal size one another and the axis (M) runs through between A_1 and A_2 . Then, let B_1B_2 and C_1C_2 be other trains each of which consists of two bodies in equal size to each of two parts of the train A_1A_2 respectively. Now let it be that both of the trains B_1B_2 and C_1C_2 are at rest and the right end of B_2 is on the M and the left end of C_1 is on the M (Fig. 1). Now, let us suppose the next scene in which both of the trains B_1B_2 and C_1C_2 begin to move towards the opposite direction at equal speed simultaneously (at t_1) and after a given time (at t_2) they shifted their initial positions to the next ones where all the three rows A_1A_2 , B_1B_2 and C_1C_2 were opposite each other (Fig. 2).

Then, it is obvious that the body B_2 has passed one A (namely the body A_2) which is equal to B_2 , while the body C_1 which is equal to the body B_2 (=1A) has passed two Bs [namely the body B_1 (1A) and the body B_2 (1A)]. Based on this setting, Aristotle insists that Zeno concluded that one A is equal

Fig. 1

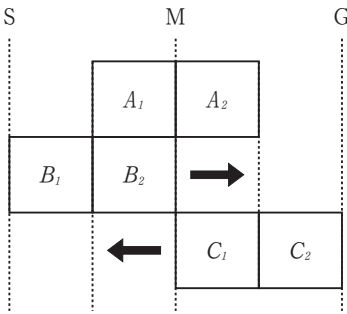
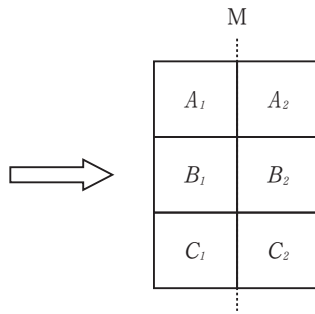


Fig. 2



to two A (namely $1A = 2A$), therefore also that ‘half the time is equal to its double.’

Is this argument, if it were Zeno’s original one, acceptable? Someone surely does not think so and abruptly may speak once for all: ‘This is not even a paradox! Only a sophism! Nowadays no one is so foolish enough to accept such a stupid talk!’ According to N. Booth, Zeno made a banal mistake and the *Stadium* is a blunder due to his ignorance of the relative motion: ‘Zeno was perhaps the first to bring theorising about time and motion into the Stadium. This in itself was a great achievement; it is not to be wondered at if Zeno, in first introducing this kind of theorising, made what appears to us to be an elementary blunder about relative motion.’⁴³⁾ However, in sharp contrast to this disappointing evaluation, M. Schofield regarded Zeno’s conclusion as a “satisfying puzzle.” He said as follows: ‘Aristotle’s own version of the argument yields a more satisfying puzzle than he allows. Baldly stated, the assumption he attributes to Zeno does look like a banal mistake. But in order to force us to the conclusion ... Zeno needs only to get us to accept the plausible idea that if a body moves past n bodies of size m , it moves a distance of mn units; simple

ZENO'S FOUR PARADOXES AGAINST MOTION

arithmetic will then show that moving mn units will take half the time of moving $2mn$ units at the same speed. Nor is this idea about the measurement of movement easily abandoned in favor of a relative theory. For if the distance a body moves is simply a function of its positions relative to other bodies, is there any absolute basis for ascribing movement to *it* at all?⁴⁴⁾

11 ZENO AND THE CHALLENGERS TO THE PARADOXES

Though not sufficiently delineated, the above mentioned will do much for the introduction into Zeno's four paradoxes. Now, I would like to start my work of digging up the layers of Zeno's paradox immediately. But, from where and how should we get on work? Leaving an aimless eye and resounding ear,⁴⁵⁾ we must have a carefully worked-out plan in advance. Otherwise we will be wrecked in the midst of a great sea of papers. The total of literatures about Zeno in status quo amounts to the volumes filling up a hovel. For a long period from the fifth century B.C. to our century, a number of people have tried to solve Zeno's paradoxes and preserved their own results. It is proverbially said that many drops make a shower. Thus, if we are excessively absorbed in carping at trifle differences between the multifarious interpretations and in adhering to giving one comment after others endlessly, we will be destined after all to become Achilles or Tristram Shandy who could not attain their objects. They are persons from whose blunders we should educe a lesson for us.

My task is to excavate Zeno's real figure, which a large quantity of surface soil covers. In order to dig up the real Zeno successfully, we must break through the bulky layers of documents and interpretations. These layers consist of four sub-layers: (1) the readings of the text, (2) the philological interpretations, (3) historical observations, and (4) the philosophical

interpretations. In our task of excavation we must pay our due attention to a focal point whereto various significant testimonies and discourses from these different sub-layers converge. This is the *topos*, in other words “common place,”⁴⁶⁾ or “common premises”⁴⁷⁾ of the “Zeno” problems. Here is the threshold from where we should begin to trench in search of Zeno’s invisible figure. Still, there are numerous literatures derived from these sub-layers. Therefore, in order to minimize references, cross-references and quotations as much as possible, we are obliged to select some suitable persons for our enterprise. This is a hard task which tends to give rise the various uncontrollable objections.

However, I dare to mark (1) Aristotle and Simplicius (for the reading of text), (2) G. Vlastos and G. E. L. Owen (for the philological interpretation), (3) T. L. Heath and Á. Szabó (for the historical observation in the field of Greek mathematics), (4) B. Pascal, H. Bergson, and B. Russell (for the philosophical interpretation) out for their positions.

(1-i) The selection of Aristotle is a supreme order. As it was said previously, there is no way to grope for Zeno’s four arguments against motion without relying on Aristotle’s report in the *Physics* Z 9, 239b5–240a18. (1-ii) And if we cannot consult with Simplicius the best commentator of Aristotle’s *Physics*, our information about Zeno might become a very poor and partial. (2-i) As one of the representatives of the Classical world in the twentieth century G. Vlastos left a series of papers on Zeno’s paradoxes, which has been evaluated by authorities as the distinguished achievements. His works on Zeno held undisputed leadership among others. (2-ii) The same can be said about G. E. L. Owen too. He made a significant contribution to the study of Eleatics. (3-i) Up to now the importance of T. L. Heath’s works in the field

ZENO'S FOUR PARADOXES AGAINST MOTION

of Greek mathematics still remains to be unchanged. It is a miracle fact. (3-ii) *Á. Szabó's* contribution to the study concerning the origin of Greek axiomatic and deductive mathematics was strongly provoked by his own study of Eleatics, especially of Zeno. (4-i) B. Pascal was, in my view, a "Jansenist Zeno" who developed an original idea of the "two contrary infinities" which tells the whereabouts of Cartesian intellect and which bears a close resemblance to Zeno's *Dichotomy* and *Achilles*. (4-ii) For H. Bergson who was one of the representatives of the Philosophical world in the early period of the twentieth century Zeno's paradox was his philosophical starting point. Throughout his philosophical life Bergson continued to return to this starting point repeatedly. However, it is an ironical fact that he himself regarded Zeno as the founder of the fallaciousness of the modern-scientific way of thinking. (4-iii) One may say that my selection of B. Russell is rather arbitrary. Yes, but, Russell's mathematical discourse about Zeno's arguments against motion, in which he uttered a famous appraisal of Zeno's four paradoxes, is so keen and suggestive that for the students of Zeno's paradoxes it became something special like a place of pilgrimage, where everyone ought to visit at least once.

Making an objection against my method one may reproach me as follows: 'Your way of doing things is nothing but to make an extravagant advertisement. Yet your real way of selection is too confined and old-fashioned.' But, I do not think so. Selecting the above mentioned persons, I intend to give signposts with reference to which we can go through a subterranean passage which leads into whereabouts of the real Zeno. Thus, the selection does not prevent to refer to many other persons who range from Homer to A. Einstein.

III THE FIRST LAYER OF ZENO'S PARADOXES

Now let me begin to dig up the first layer of Zeno's paradoxes and make investigation into Zeno's strategy which lies behind the surface of his four arguments against motion. What was Zeno's real intention in the four arguments against motion? To answer this question we must take up afresh each of the four arguments again and examine in detail the property of the target which they aim at and the interrelationship between them.

1 THE *DICHOTOMY*

The *Dichotomy* says: 'An object in motion is not in motion, because a thing in motion must reach the half-way point before it gets to the end.' In this argument Zeno presupposes that the motion which reaches 'the half-way point' had to have been completed before it gets to the end. Pay attention to a fact that even the motion getting to the end is, by a tacit consent, presupposed to have been completed.

This is confirmed by Aristotle's another reference to the *Dichotomy* at *Phys.* Θ, 263a4 where he says:

'The same method should also be adopted in replying to those who ask, in the terms of Zeno's argument, whether we admit that before any distance can be traversed half the distance must be traversed, that these half-distances are infinite in number, and that it is impossible to traverse distances infinite in number.'

On the other hand Simplicius in his comments on the above quoted

ZENO'S FOUR PARADOXES AGAINST MOTION

Aristotelian context says as follows:

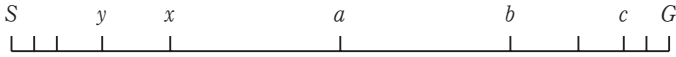
‘The argument of *Zeno*, to which he [Aristotle] now refers, was as follows: If there is motion there will be something which *has traversed* (διεληλυθός) an infinite number of positions in a finite time; for, since the process of dichotomy can continue infinitely, in every continuum there will be an infinite number of halves owing to every part of it having a half. Therefore *a body which has moved* (τὸ κεκινημένον) over a finite distance will *have traversed* (διεληλυθός) an infinite number of halves in a finite time, that is, in the time which it took to traverse the finite distance in question.’⁴⁸⁾

Each of the italicized expressions is in the *present perfect* tense. The fact makes it obvious that Simplicius here regards the *Dichotomy* as an argument which presupposes an infinite series of *completed* motion.⁴⁹⁾

To sum up, the *Dichotomy* proceeds as follows: In the first place we are induced to imagine an admirable runner like swift Achilles who *has* finally *overtaken* Hector ‘when for the fourth time they were come to the springs,’⁵⁰⁾ who *have traversed* successfully the entire course. Now in the first stage the runner who reached successfully the goal is wiping his face free of perspiration. But in the second stage, the runner is accused of a crime by Zeno the judge for “the failure of the infinite halves running.”

The gist of accusation is as follows: ‘The accused states that he completed his running along the course. But it must be false, because if he could have traversed the entire course of stadium he should be able to count or touch as often as each of the half-way points as he comes to it one by one. But it is

Figure 3



beyond doubt that there are an infinite number of the half-way points in the course, because it is a continuum and every continuum has an infinite number of halves.⁵¹⁾ And it is obvious that nobody can count or touch an infinite number of the half-way points one by one. Therefore, it is obvious also that he could not traverse the entire course of the stadium. Thus, the runner who is alleged to have traversed the entire course of stadium has not really traversed the course. Therefore also a body (which is alleged to be) in motion is not in motion.’

It is difficult to identify which type of half-way running Aristotle had in his mind when he referred to the *Dichotomy* saying that: ‘An object in motion must reach the half-way point *before* it gets to the end.’ What remains to be unclear is the implication of the phrase ‘*before* (πρότερον ... ἤ)’.

Let *S* be the starting point and *G* be the goal. Then, the runner must first reach the half-way point *a* *between* *S* and *G* before he gets to *G*. But, thereafter, towards which direction does he move? There are two possibilities: (1) the runner moves backward; then, he must first reach the half-way point *x* *between* *S* and *a* before getting to *a*, and so on; or (2) the runner moves forward; then, he must reach half-way point *b* *between* *a* and *G* before getting to *G*, and so on.

Philoponus took the former interpretation and said:

ZENO'S FOUR PARADOXES AGAINST MOTION

‘If anything moves along a given finite straight line, it must, before moving along the half of the whole, move along a quarter of it, and before a quarter an eighth, and so on *ad infinitum*; for the continuum is infinitely divisible.’⁵²⁾

Recognizing the *Dichotomy* as a kind of an “infinite regress” argument Ross also took sides with the former interpretation.⁵³⁾ It is provably a natural interpretation. But, the issue is controversial among scholars. For example, with some legitimate reasons G. Vlastos took sides with the latter interpretation to which however I will refer later and present some objections.

Be that as it may, this issue does not pose the essential problem. The fundamental problem which the *Dichotomy* brings forward has its essential relevance with motion “on the way from x to y ,” in other words, with the concept of “between-ness;” i.e., the “extension” and its “denseness.” A. Grünbaum tried to construe the *Dichotomy* and the *Achilles* as ‘offering a *reductio ad absurdum* of the *denseness* of physical time and of motion but not as denying their existence (“reality”).’⁵⁴⁾ This is a keen insight to which I will later develop an argument.

2 THE ACHILLES

Aristotle himself calls Zeno’s second argument against motion simply “the so-called *Achilles*” and treats it as essentially the same as the *Dichotomy*:

‘ἔστι δὲ καὶ οὗτος ὁ αὐτὸς τῷ διχοτομεῖν, διαφέρει δὲ ἐν τῷ διαιρεῖν μὴ δῖχα τὸ προσλαμβάνομενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβάνεσθαι τὸ βραδύτερον συμβέβηκεν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταῦτὸ τῇ διχοτομί-

α—έν ἀμφοτέροις γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρας διαιρουμένου πῶς τοῦ μεγέθους· ἀλλὰ προσκεῖται ἐν τούτῳ ὅτι οὐδὲ τὸ τάχιστον τετραγωδημένον ἐν τῷ διώκειν τὸ βραδύτατον—ὥστ’ ἀνάγκη καὶ τὴν λύσιν εἶναι τὴν αὐτήν.’

(‘This argument is essentially the same as that depending on dichotomy, but differs in that the successively given lengths are not divided into halves. The conclusion of the argument is that the slower runner⁵⁵⁾ is not overtaken, but it proceeds on the same lines as the dichotomy argument, since in both, being divided the distance in a given way, it is concluded that the goal is not reached: only in the *Achilles* a *dramatic effect* is produced by saying that not even the swiftest will be successful in its pursuit of the slowest and so the solution of it must be the same.)⁵⁶⁾

It is a notable fact that Aristotle in the above quotation refers to the “dramatic effect” (τετραγωδημένον) of Zeno’s second argument. But, the saying seems to be somewhat beside the point, if we could not know Achilles’ rival. Who is Achilles’ rival?

According to Simplicius the commentator, Achilles’ rival is a tortoise. Simplicius insists that Zeno’s second paradox against motion, the *Achilles*, was so called “because of the introduction into it of Achilles, who cannot possibly overtake the tortoise he is pursuing.”⁵⁷⁾ But, why does a tortoise make its appearance all of a sudden? It is, according to H. D. P. Lee, because of its dramatic and sensational effect.

Referring to the word-meanings of ‘τετραγωδημένον’⁵⁸⁾ Lee said as follows: ‘the introduction of Achilles and the tortoise in this [second argument], of the flying arrow in the next argument, and of the stadium in the last give them a

ZENO'S FOUR PARADOXES AGAINST MOTION

certain *vivid* and *dramatic* quality, of which *Aristotle evidently thoroughly disapproved, but which seems to have been very characteristic of Zeno's reasoning.*' [italic by Yamakawa]⁵⁹⁾

But, what does Aristotle's disapproval in this context exactly mean? On this matter W. D. Ross said as follows:

'In fact, practically the only detail that reaches us independently of him [Aristotle] is that Achilles' rival in Zeno's parable was a tortoise. This comes to us from the Greek commentators on Aristotle. The reference to the tortoise comes also from Plutarch, but in Plutarch the tortoise is matched not against Achilles but against "the fast horse of Adrastus,"⁶⁰⁾ so that possibly Achilles' rival was not a tortoise at all, and two independent stories may have got mixed up in the familiar title.'⁶¹⁾

I do not know, of course, what kind of the opponent party of Achilles Ross had in mind by his saying that 'possibly Achilles' rival was not a tortoise at all.' For my part, however, contrary to Ross' conjecture, I surmise that Zeno himself regarded originally a tortoise as Achilles' opponent party.

It is a definite fact that "the swiftest runner" (τὸ τάχιστον) refers to Achilles the hero. Then, why is "the slowest runner" (τὸ βλαδύτατον) a tortoise?

- (1) In Homer's Iliad we can witness an imposing figure of Achilles, who is pursuing at full speed Hector as though he were 'a falcon (κίρκος) ... the swiftest of winged things (ἐλαφρότατος πετεινῶν).'⁶²⁾
- (2) Homer in the above context likens Hector to "a trembling dove" (τρήρων πέλεια).⁶³⁾ Therefore, a pair of 'Achilles : Hector' in this context is

likened to the pair of “a falcon : a trembling dove.’

- (3) But, the supposition that “the slowest runner” in Zeno’s context is “Hector” is out of question. Hector is never a slowest runner. At *Iliad*, XXII. 164–166 a footrace between two heroes is depicted as follows: ‘These two heroes circled thrice *with swift feet* (καρπαλιμοισι πόδεσσι) about the city of Priam.’ Hector is not a tardy runner, but Achilles’ capable rival. ⁶⁴⁾

Who is, then, the slowest runner? According to Simplicius it was a tortoise (χελώνη). But, why is a tortoise Achilles’ rival? Why cannot Achilles’ opponent party be a trembling “dove” or a “fawn”? Is it not by any chance a forgery by Simplicius?

The last supposition however comes into collision with evidences. In the first place, Simplicius could presumably consult with a copy of Zeno’s original book⁶⁵⁾ or at least he could resort to ancient reports of Zenonian arguments from a good source such as one of Theophrastos.⁶⁶⁾ Second, there is a fact that, before and apart from Simplicius, Themistius too refers to the “tortoise” as Achilles’ competitor.⁶⁷⁾ In addition to this, third, Simplicius himself refers to Hector who is never overtaken by Achilles.⁶⁸⁾

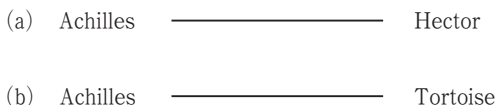
‘τῷ δὲ ἐπ’ ἄπειρον ἔλαττον ἄλλο ἄλλου διάστημα λαμβάνειν διὰ τὴν ἐπ’ ἄπειρον τῶν μεγετῶν τομὴν, οὐ μόνον Ἐκτωρ ὑπὸ τοῦ Ἀχιλλέως οὐ καταλήφθησεται, ἀλλ’ οὐδὲ ἡ χελώνη.’

(‘And so by taking distances decreasing in a given proportion ad infinitum because of the infinite divisibility of magnitudes, we arrive at the conclusion that not only will Hector never be overtaken by Achilles, but not

ZENO'S FOUR PARADOXES AGAINST MOTION

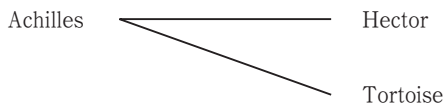
even the tortoise.)

In this context of Simplicius' utterance, it is obvious that Simplicius has in mind two parties of the running match; that is, (a) a party of Achilles and Hector and (b) another party of Achilles and a tortoise.

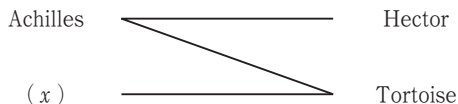


The diagram (a) is a familiar one for the reader of Homer's *Iliad*. But, (b) is unfamiliar and unexpected one, which has, as Aristotle said, a dramatic effect and might caused a sensation among people. Then, who is the inventor of the diagram (b)?

It is a noteworthy fact that the above pair of (a) and (b) is a compressed expression derived from the following diagram:



And this diagram seems to hide its behind the third party whose one competitor is tortoise, but another competitor is unknown "x":



Someone has substituted "x" for "Achilles." Then, who is "x" and who was

the author of a pair “Achilles and Tortoise”? I surmise that “x” is a “hair” and the original author of “Achilles and Tortoise” is Zeno of Elea.

Before the problem in question is settled, we must take up another problem, which invites us to adopt one of the alternatives; either (1) Zeno produced an argument against motion, the title of which was originally “Achilles and Tortoise.” or (2) Zeno did not give a particular title to the second argument, whereas in his context of argument he assigned a part of “the slowest runner” to a tortoise and “the swiftest” to Achilles.

Reviewing the opening scene of Plato’s *Parmenides*, we find that the latter alternative appears to hit the mark. When the reading of a part of Zeno’s book finished, Socrates at 127d-e asked to hear the “hypothesis” of “the first argument” again. It is a noteworthy fact that Socrates does not make mention of their titles. It seems, therefore, they had originally no titles. On the other hand, the style of hypothetical argument presented by Zeno in the same dialogue is indeed reminiscent of Zeno’s genuine arguments against “plurality” preserved by Simplicius.⁶⁹⁾ They are very abstract and dialectical. But the matter does not always eliminate the case that Zeno appealed to people’s imagination.⁷⁰⁾ Gorgias’ ‘On What Is Not or Concerning Nature’ presents the parallel case.⁷¹⁾ In the midst of developing very difficult and abstract argument Gorgias could effectively use some vivid words such as “a man flying,” “chariot running over the sea,” etc.⁷²⁾

Now let’s return to the above mentioned question: ‘Why did Zeno adopt a “tortoise” as the competitor of Achilles? To this question I have no other hypothesis than that Zeno had presumably in mind Aesop’s fable “Tortoise and Hare” or one of its variations.

But, did Zeno really know this fable? Yes, I think so. The fable in question

ZENO'S FOUR PARADOXES AGAINST MOTION

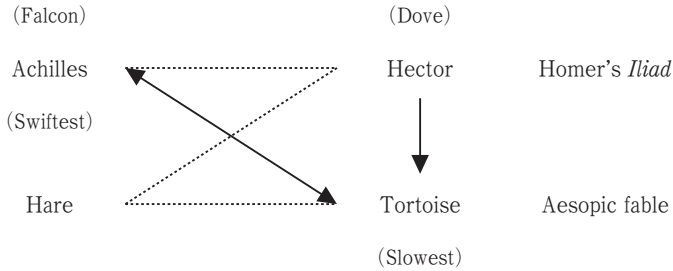
dates back at least to Zeno's contemporary age. For it is included in the *Augustana Recension* whose primary source is possibly the *Aesopia*, the first collection of Aesopic fables published by Demetrius of Phalerum in the end of the fourth century B.C. or in the beginning of the third century B. C.⁷³⁾ Demetrius of Phalerum compiled the Aesopic fables which were in circulation for a long time among Greeks and which was published 'as a handbook of materials intended primarily for the use of writers and speakers.'⁷⁴⁾

It is a well-known fact that Aristophanes utilized often Aesop's fables in his dramatization.⁷⁵⁾ Socrates in prison also was concerned with versification of Aesop's fables in his last days.⁷⁶⁾ Similarly for Hesiod, Sophocles, Herodotus, Xenophon, Plato, and Aristotle too the animals like eagle, beetle, fox, lark, hare, and tortoise etc. making their appearances in the Aesopic fables were, in greater or lesser degree, familiar ones. Thus, there is nothing to be wondered about a fact that Zeno himself also was familiar with Aesopic fables.

However, you may still ask me stubbornly: 'Is it sure that Zeno was familiar with the fable "Tortoise and Hare"?' I do not know exactly whether it was the case or not. But, it is sure that Menedemus the contemporary rhetorician of Plato was well informed of this fable. Once quoting the lines from the *Omphale*, a satiric drama of Achaëus the tragicus, Menedemus attacked his political opponents,:

'Ere long the swift is overtaken by the feeble,
And the eagle (ἀετός) by the tortoise (χελώνη).'⁷⁷⁾

Note that this is another version of "Tortoise and Hare." The 'eagle' corresponds to a 'falcon' in the above-mentioned Homeric passage.



Thus, it is obvious that the Zenonian analogy of ‘The Swiftest : The Slowest = Achilles : Tortoise’ must have been very suggestive and impressive for everyone who has been already well versed in the Homeric analogy: ‘Achilles: Hector = Falcon : Dove’ as well as Aesopic fable ‘Tortoise and Hare’ or its variations.

As you know well now, to win the hearts of the people by arousing their vivid imagination was Zeno’s important strategy.

Now, if the above mentioned hits somehow the mark, I hope, the following imaginary dialogue between Achilles and Tortoise also may be regarded as a not so bad revision of Lewis Carroll’s witty tale. It is a sequel to the footrace of Achilles and Tortoise which touches the core meaning of Zeno’s second argument against motion.⁷⁸⁾

Suppose the swift runner Achilles could overtake a tardy tortoise at a spot n , and now sitting triumphantly on the tortoise’s back, he arrogantly says as follows:

Achilles: ‘I have conquered you. Now, you must admit your loss!’

Tortoise: ‘Hmmm ... but, I cannot believe my ... rather unpleasant ... state of

ZENO'S FOUR PARADOXES AGAINST MOTION

affairs. Disclose your trick, Oh, no! ...but strategy, or way of running, so I may concede my loss.'

Achilles: 'Well, listen to my words carefully and satisfy yourself of your miserable situation. My way of running was a splendid one. In the first step, I covered a distance greater than a half of the distance which could be shown as a length between the starting point and your final spot here. And in the second step, I covered a distance greater than a half of that which was left. Thus repeatedly employing such a strategy, I finally overtook you, and now I am resting comfortably on your back. Do you understand?'

Tortoise: 'Oh, I see. Your way of running was really based on the Axiom of Archimedes.'

Achilles: 'Not 'Archimedes', but Euclid's *Elements* X. Prop. 1. In any way you are not enlightened about the history of mathematics.'

Tortoise: 'Don't mind such a trifling matter. Well, I recommend you to listen to Zeno's argument. He says: "Even the slowest runner, when in motion, can never be overtaken by the fastest, for, inevitably, the pursuer must first reach the point from which the pursued started; it means that the slowest runner will always remain ahead." If your way of running were Zeno's, I could never be overtaken by you, since he assures us that the two distances covered by you and me were really incommensurable.'

The *Achilles* is, according to Aristotle, 'essentially the same as that depending on the *Dichotomy*, but differs in that the successively given lengths (τὸ προσλαμβανόμενον μέγεθος) are not divided into halves.' Therefore, it is

beyond doubt that in this argument too the motion is viewed in the aspect of the perfect tense and its subject is the problem of the between-ness and the denseness of extension. But, the *Achilles* can be regarded also as a representation of the process of successive subtraction between two incommensurable quantities. Here in this context, with prominent free use of *reduction ad absurdum* and possibly the theory of “even and odd” which was employed in Euclid’s *Elements* VII and IX, the incommensurability between two distances traversed by Tortoise and Achilles becomes afresh to be actualized in connection with the method of “successive subtraction” (ἀνθυφαίρεσις or ἀνταναιρέσις).⁷⁹⁾

The fact will introduce us to the mathematical dimension of Zeno’s second argument.

3 THE ARROW

The *Arrow* might immediately remind the reader of the *Odyssey* the famous scene in the *Odyssey* XXI 76–423, where Odysseus shoots an arrow through all twelve of the axes. Arousing such a thrilling feeling in the public, Zeno in the next stage declares that the arrow passing through all the twelve sockets of the axes is *toties quoties* in its passing through each socket and consequently also always at rest during its flying.

Unfortunately, the *Arrow*’s text is corrupt and incomplete. It reads ‘ἤρημει πᾶν ἢ κινεῖται’ at 239b5–6. But this reading appears to be inconsistent with succeeding sentence: ‘ὅταν ἢ κατὰ τὸ ἴσον ...’ Following Themistius’ reading ‘ἤρημει πᾶν’ Zeller ejected ‘ἢ κινεῖται’ on the ground that Zeno’s concern here is to give the definition of “rest.” Burnet, Cornford, Ross and others too followed this reading.⁸⁰⁾

ZENO'S FOUR PARADOXES AGAINST MOTION

However, I think, we should follow Diels' emendation.⁸¹⁾ Preserving 'ἡρεμεῖ πᾶν ἢ κινεῖται,' he inserted immediately after this <οὐδὲν δὲ κινεῖται>. It seems to be legitimately acceptable, because we read 'ἢ κινεῖται' or its equivalent in Simplicius' and Philoponus' texts.⁸²⁾ As Lee insists, there is no manuscript authority for omitting them. Hence, we should suppose not that 'ἢ κινεῖται' is an insertion, but that original words such as 'οὐδὲν δὲ κινεῖται' dropped out.⁸³⁾

However the adoption of Diels' reading appears to come immediately into collision with D. Ross' apparent reasoning. Ross insisted that 'ἢ κινεῖται would be in point only if Zeno had argued disjunctively ...'⁸⁴⁾ But, without fail Zeno here argues deliberately disjunctively. For Zeno's premise 'ἡρεμεῖ πᾶν ἢ κινεῖται' should be regarded as a specimen of the Parmenidean law of the excluded middle.⁸⁵⁾

In order to confirm the cardinal point of Zeno's logical manoeuvre in the *Arrow*, let me turn a while to the conclusions of the *Arrow* I and II:

The *Arrow* I The arrow in motion is motionless. (1)

The *Arrow* II The arrow in motion is at rest. (2)

However, the above (1) and (2) in lump have the same import:

The arrow which is in motion is not in motion. (3)

And the above (3), after all, entails that

The same arrow is and is not in motion at the same time. (4)

The above (4) is a plain absurdity which violates the law of contradiction. Therefore, we must approve that the negation of (4) is true:

It is not the case that the same thing is and is not in motion at the same time. (5)

Then, the above (5) entails the following:

The same thing is either in motion or is not in motion at the same time. (6)

And the above (6) is substantially equivalent to the statement that ‘everything is either at rest or in motion (ἡρεμεῖ πᾶν ἢ κινεῖται),’ which is nothing but Zeno’s major premise in the *Arrow* I. Therefore, in spite of Ross’ apparent judgement, Zeno in the *Arrow* argues in reality disjunctively.

By the way, the above (6) is, as we can appreciate immediately, just a variation of the following more general principle:

The same thing is either in the place where it is or in the place where it is not at the same time. (7)

In truth Zeno’s fundamental premise against motion is this exclusive disjunction. And we can confirm the case in [1] *DL*, IX. 72 (*DK*, B4), [2] Epiphanius, *Adv. Haer.* III. 11 (Diels, *Dox.* 590. 20), and [3] Sextus Empiricus, *Outlines of Pyrrhonism*, III. 71.⁸⁶⁾

[1] A thing in motion is not in motion either in the place where it is or in the place where it is not.⁸⁷⁾

[2] A thing in motion is in motion either in the place where it is, or in the

ZENO'S FOUR PARADOXES AGAINST MOTION

place where it is not; and it is not in motion either in the place where it is or in the place where it is not; therefore nothing is in motion.⁸⁸⁾

[3] If a thing is in motion, it is in motion either in the place where it is or in the place where it is not. But it is not in motion in the place in which it is, for if it is in it, it remains in it; nor yet it is in motion in the place where it is not; for where a thing is not, there it can neither effect nor suffer anything. Therefore nothing is in motion.⁸⁹⁾

The testimony [1] exactly corresponds to each of the second half of [2] and [3] respectively. Therefore, if the fragment [1] preserves a genuine part of Zeno's *Arrow*, we may regard each of [2] and [3] as a fairly trustworthy replica of Zeno's *Arrow*.

Now, it is a noteworthy fact that Epiphanius and Sextus Empiricus begin their reports with the following words:

[2] A thing in motion is in motion either in the place where it is, or in the place where it is not.⁹⁰⁾

[3] If a thing is in motion, it is in motion either in the place where it is or in the place where it is not.⁹¹⁾

Except for a minor difference in their syntactical construction, both of [2] and [3] bear a close resemblance between them. So we may suppose that both of them are substantially trustworthy replicas of Zeno's *Arrow* and the logical form which they hold in common can be legitimately attributed to Zeno's original argument, which may be strictly reformulated as follows:

A thing in motion is in motion either in the place where it is now or in the place where it is not now. (8)

Note a fact that in the above (8) there is an illegitimate disjunct: ‘a thing in motion is in motion in the place where it is not now.’ This disjunct is of course false; you in a class room here and now cannot be singing a song in a broadcasting station far from here 2 km where you are not now; similarly, an astronaut who is now under training in NASA, cannot play golf on the surface of the moon at the same time. Therefore, in reality, there remains just one disjunct, namely ‘in the place where it is *now*.’⁹²⁾

Then, what does a definitive word “now” in this context mean? We must remember that Aristotle in the *Arrow* I criticized Zeno’s argument as follows:

This is false; since time is not composed of a *number of indivisible now* any more than any other magnitude is composed of indivisibles (τοῦτο δ’ ἔσται ψεῦδος· οὐ γὰρ συγκεῖται ὁ χρόνος ἐκ τῶν νῦν τῶν ἀδιαιρέτων, ὥσπερ οὐδ’ ἄλλο μέγεθος οὐδέν).

But, in so expressing his view, which kind of “indivisible now” did Aristotle have in mind? We must be careful to distinguish between Zenonian indivisible “now” in the *Arrow* I & II and the Aristotelian one. Aristotle defines the “now” as ‘the limit up to which the past has run, none of the future being this side of it, and also as the limit from which the future runs, none of the past being that side of it.’⁹³⁾ He insists that in such an indivisible now anything neither can be in motion nor at rest. Such an indivisible now is, according to Aristotle, is not time, but only a limit of time.⁹⁴⁾ It has no extension wherein a

ZENO'S FOUR PARADOXES AGAINST MOTION

thing can be in motion or at rest. Hence, such an “indivisible now” never authorizes one to infer the alleged Zeno’s conclusion that ‘the arrow in motion is at rest.’ And yet, it is notable, Aristotle in the *Arrow* II insists that the conclusion in question follows from Zeno’s assumption of the “indivisible now.”⁹⁵⁾ Therefore, it is obvious that the indivisible “now”, which Aristotle attributes to Zeno in the *Arrow* II, is different from Aristotelian one as a limit between the past and the future.

Therefore it is considerably clear that Aristotle here attributes to Zeno the assumption of “atomic now,” i.e. of the “indivisible present” with minimum magnitude. The case can be ascertained by a fact that Aristotle says that time is not composed of a number of indivisible now *any more than any other magnitude is composed of indivisibles.*’ The statement may be regarded as a lemma of his conclusion at *Phys.*, 232a23–25 where it is said that ‘it is impossible for anything continuous to be composed of indivisible parts.’⁹⁶⁾ There Aristotle argues that if length and motion are indivisible, it is necessary that ‘time also be indivisible, i.e. be composed of indivisible nows’ (συγκείσθαι ἐκ τῶν νῦν ὄντων ἀδιαίρετων); but, according to Aristotle, every motion is divisible; therefore time must be also divisible.⁹⁷⁾ The reasoning however tacitly presupposes that the indivisible time in question has a magnitude and divisible.

Similarly Aristotle at 233b15ff argues that ‘neither a line nor a surface nor in fact anything continuous can be indivisible’ and proceeds to demonstrate the case. The very core of the demonstration consists in that ‘the opposite assumption implies the *divisibility of the indivisible.*’ That is to say ‘the indivisible will be divisible, and that which has no parts will be passed over not in an indivisible but in a greater time.’⁹⁸⁾ What is called here “which has no part” is not the limit between the past and the future but the indivisible time-dimension.⁹⁹⁾

Thus Aristotle here without fail presupposes that the indivisible “now” has a magnitude.¹⁰⁰⁾ Therefore it is obvious also that Aristotle here has in mind the atomic “now.” In the *De Generatione et Corruptione* Aristotle similarly tries to refute an argument ‘which is believed to establish the necessity of atomic magnitude.’¹⁰¹⁾ The theory, according to him, erroneously assents to that there must be indivisible bodies and magnitudes, because of a fact that the infinite division of a magnitude involves an absurdity.¹⁰²⁾

Needless to say, such an atomistic conception of magnitude may be regarded as a response to Zeno’s refutation against the plurality,¹⁰³⁾ i.e. a response to the antinomy of the so-called Zeno’s metrical paradox of extension.¹⁰⁴⁾ According to Simplicius Zeno developed an argument which ‘was thought to prove by means of dichotomy that what is, is one only, and accordingly without parts and indivisible.’¹⁰⁵⁾

For, he argues, if it were divisible, then suppose the process of dichotomy to have taken place: then either there will be left certain ultimate magnitudes, which are minima and indivisible, but infinite in number, and so the whole will be made up of minima but of an infinite number of them; or else it will vanish and be divided away into nothing, and so be made up of parts that are nothing. Both of which conclusions are absurd. It cannot therefore be divided, but remains one.

This is a destructive dilemma that leads anyone who adopts either the one horn of indivisible and infinite “minima” or the other horn of “parts that are nothing” into self-contradiction. An important aspect of Zeno’s argument against plurality consists in the way of Parmenidian juxtaposition between

ZENO'S FOUR PARADOXES AGAINST MOTION

what has dimensions and what is dimension-less.¹⁰⁶⁾ Aristotle's formulation of the *Arrow* corresponds to this Zenonian structure and refers to the Zenonian first horn; he puts the supposition of infinite and indivisible time which Diodorus Cronus revived in later period in question.¹⁰⁷⁾ Diodorus Cronus argues within the framework of Zeno's metrical paradoxes. Originally he had nothing to develop his novel contrivance. As we have confirmed Diodorus Cronus invoked literally Zeno's disjunctive inference to refute the possibility of motion. The idea of the atomic "now" indwelled originally in Zeno's conception of a version of commonsensical conception of time, let alone the well-polished Aristotelian conception of the limit between the past and the future.¹⁰⁸⁾

The atomic "now" is never a castle in air which some scholars' 'neat fantasy' produced.¹⁰⁹⁾ It is just an original device due to Zeno himself. For Zeno's metrical paradoxes had a contrivance to suspend Parmenides' adversaries in midair between both extremities of continuity and atomicity.

Here, however, as Barnes and many other scholars did, you may object to me saying that where is 'evidence that anyone prior to Zeno had entertained the atomistic theory he is imagined to be attacking.'¹¹⁰⁾ But this is quite ridiculous. First, you should be aware of the absurdity of a dogma that a new hypothesis should coincide with a ready-made one. As Feyerabend in *Against Method* said, such a belief is plainly irrational, because it hinders the development of a new theory.¹¹¹⁾ Second, it is not necessary to specify the names of Zeno's adversaries. You should rather be aware of a technical fault of your way of arresting the criminal. Before identifying in a hit-or-miss manner an individual with a criminal, you should have a theory of the criminal investigation which permits you squeeze a particular person to the last penny. This was, I

believe, Zeno's method.

By the way, it is noteworthy that P. Curd modestly foresaw a new figure of Zeno. According to her Zeno 'is surely an Eleatic,' but 'also a serious philosopher' who raised the problems about 'the divisibility of a basic entity' 'with which Plato and Aristotle will both wrestle.'¹¹²⁾ Well, but, we should keep in mind a fact that not only both of Plato and Aristotle but also Aaxagoras, Empedocles, Leucippus and Democritus also had to wrestle with the problems of divisibility and indivisibility within a framework of dragnet which was originally spread by Zeno who aimed at catching all the pluralists including the prospective adversaries in future. Then, can you specify and name a prospective criminal in future?

So far we have confirmed that the *Arrow* begins with presupposition of the indivisible "now" as atomic one. The issue naturally extends to the interpretation of "κατὰ τὸ ἴσον." Aristotle believed that spatial magnitude, motion and time are isomorphic each other; he believed that if motion is divisible, the other two are also divisible; and if the motion is indivisible, the other two are also indivisible.¹¹³⁾ Thus Aristotle says: 'For because magnitude is continuous, motion is also continuous, and time because of motion.'¹¹⁴⁾ Therefore, we should naturally consider that Aristotle in the *Arrow* too presupposes such an isomorphism between them. Then, we should approve of Cornford's remark at the note 2 in LCL to the reading of ρϕ (ἔστι δ' αἰεὶ τὸ φερόμενον ἐν τῷ νῦν κατὰ τὸ ἴσον) as a legitimate one. He said as follows: 'At every moment the moving thing is occupying the moment (of the time occupied by its whole movement) which corresponds to the space equal to its dimensions,' provided that 'the time is supposed to be made up of a row of successive indivisible moments corresponding, one to one, with the row of successive positions

ZENO'S FOUR PARADOXES AGAINST MOTION

Fig. 4

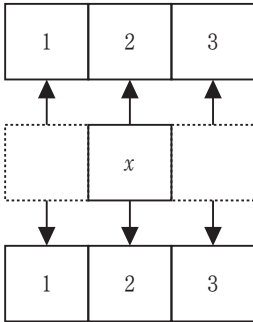
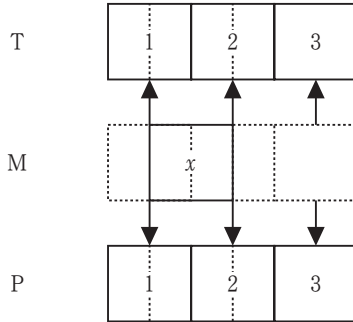


Fig. 5



occupied by the body.¹¹⁵ The words permit us to draw the following figure:

The row *T* consists of a series of the atomic nows 1, 2, 3, ...; *M* shows a series of the successive movement which has to correspond one by one to a series of the indivisible places 1, 2, 3, ... of the row *P*; *x* is a pointed head of an arrow which must be regarded as an atomic magnitude in motion. Then, the figure 4 shows the location of *x* at atomic now₂ which strictly corresponds to the indivisible place₂. But it should be the case that at the immediately before atomic time₁, *x* was located on the place₁. Then, how could *x* travelled from place₁ to place₂? By no means, if you mean a continual locomotion by the word “travelling.”

You cannot suppose a case such that the figure 5 depicts, where time and space each of which is to be indivisible must be divided, therefore also *x* itself according to the isomorphic principle. So, you are necessitated to say that ‘*x* jerks.’ It is irrational to ask once again when this jerk does happen, because there is no other time than the atomic now. At this juncture we may listen to Bertrand Russell’s words [Note a fact that what Russell calls “instant” is nothing but the atomic “now.”]:

At each of the thousand instants, the arrow is where it is, though at the next instant it is somewhere else. It is never moving, but in some miraculous way the change of position has to occur between the instants, that is to say, not at any time whatever.¹¹⁶⁾

Now let me return to our initial concern and let me schematize the logical structure of the *Arrow* which underlies the above quoted the *Arrow* I and II in order to discern the Eleatic marks therein:

1. Everything is either in the place where it is or in the place where it is not.¹¹⁷⁾
2. Everything is either in motion or not in motion.
3. If an arrow is in motion, then it is in motion either in the place where it is or in the place where it is not.
4. If it is in the place where it is, then it is in the place equal to itself¹¹⁸⁾ and remains¹¹⁹⁾ to be in the place where it is. Hence it is not in motion.
5. The alternative supposition that an arrow is in the place where it is not now is logically absurd. Hence its motion is unconceivable.
6. If an arrow is not in motion, then it is neither in motion in the place where it is nor in the place where it is not.
7. If it is in the place where it is, then it is in the place equal to itself and remains to be in the place where it is. Hence the arrow, again, is not in motion.¹²⁰⁾
8. The supposition that the arrow is in the place where it is not now is logically absurd.
9. It is impossible that the same thing is and is not in the same place at

ZENO'S FOUR PARADOXES AGAINST MOTION

the same time.¹²¹⁾

Therefore, a thing in motion is not in motion.

It is an interesting fact that we can discern in the above schematization of Zeno's *Arrow* a number of Eleatic marks; especially his free use of the law of the excluded middle which Parmenides habitually used in his defence of "what is" and negation of "not being."¹²²⁾

Relying on Parmenidean way of thinking, Zeno was likely at pains to construct a proposition, the subject of which is directly contradicted with its predicate: 'A is not-A.' Pay attention to a fact that A, the initially given subject, is a vivid perceptual object like a runner on the course, Achilles and a tortoise, and a flying arrow sailing across the midair. In other words, they are plain things with which everybody is well acquainted in their everyday life. But, in the next stage, in the part of predication, he will show you the invisible mechanism or circumstances where the runner is necessitated to complete his infinite half-way running or where a flying arrow must eternally be frozen. Thus, while the object occupying the position of the subject is a visible thing, one which you witness in the part of predication is an invisible thing.

In the case of the *Arrow*, however, there is another point which we should bear in mind. Aristotle referred to the *Arrow* two times. And in the second time when he mentions to the *Arrow* II he introduces it as 'the third,' in spite of his previous reference to the *Arrow* I at *Phys. Z* 9., 239b5-14. This fact suggests that the ordering of the argument originates from Zeno himself. For Zeno the *Arrow*'s position among the other three arguments logically had to be "the third."¹²³⁾ The *Arrow* so considered is, in my view, a finishing blow against those who were damaged by Zeno's preceding two arguments against motion,

i.e., the *Dichotomy* and the *Achilles*. He anticipated that they will cling to the “now” in order to recover their lost territory. To catch them with one throw Zeno spread a dragnet, i.e. the *Arrow*. Zeno surmised that they will persistently insist as follows: ‘Even if there is something like truth in what Zeno says, it is beyond doubt that the motion *is* real and that a moving body is in motion *now*, in this *instantaneous moment*.’

Rejecting this claim flatly the *Arrow* declares that: ‘what is in motion is always in a given now occupying a place equal to itself. Therefore, the arrow in motion is not in motion.’ In other words Zeno argued as follows:

‘An arrow, if it were to fly, will be helped fly *now* in *the threshold of indivisible moment*. For if the moment itself is divisible into infinite parts, it would have a series of infinite “halves” because of which again the impossibility of motion will be concluded. Therefore, if an arrow is *in* flying, it is in flying *now in this indivisible moment*, occupying a place where it *is*. Thus, the arrow in flying now occupies a place equal to itself; and as you know well, everything occupying a place equal to itself must remain to be in a place where it is now and does not change its own place; and nothing can be in motion without changing its place where it is now. Therefore, the flying arrow is not flying.’

Zeno will go on to address you: ‘you may shout, pointing to a flying *arrow*:

“Behold, an arrow is now flying!” Well, however, are you really pointing to a *flying* arrow? On the contrary, is it not the case that you are pointing to an object which is in the place whereto you point to? Then, it is in the

ZENO'S FOUR PARADOXES AGAINST MOTION

place where it is now; and hence it is at rest and is not in flying. For when you point to the arrow, it must *be* in the place whereto you point now. Or is it possible to point to an arrow being in a place where it is not now? If you insist persistently that there can be such a case, you are obliged to explain how you can insist to point to an object which is not now. Thus you must at all events admit a fact that the alleged flying arrow is in a place equal to itself and is not in another place where it is not now. Therefore, it is not in motion. And in every moment during its flying the arrow always occupies a place equal to itself. Therefore, the flying arrow is always at rest.'

Thus, in place of a flying arrow a stationary arrow makes its appearance.

4 THE *STADIUM*

The *Stadium* is, in my opinion, to be legitimately regarded as a synthetic argument which occupies its central position among other preceding arguments against motion; the *Dichotomy*, the *Achilles*, and the *Arrow*. The *Stadium* embodies various difficulties deduced from those three preceding arguments and synthesizes their apparently absurd conclusions, which may remind us those apparent and incomprehensible conclusions that Parmenides the leading character in Plato's *Parmenides* deduced from the eight hypotheses on the Unity or the One.¹²⁴⁾ But, before going into the elucidation of it, in the first place, I want to deal with a problem about the term "bodies" (ὄγκοι), which the text of the *Stadium* refers to. For the *Stadium* is presented to us as an argument about 'the equal "bodies" (ὄγκοι) moving from opposite directions alongside the equal bodies in the stadium—the ones from *the end of the stadium*, the

others from the *middle*—with equal speeds.’

The text (Aristotle, Phys, Z9, 239b33) runs as follows:

Τέταρτος δ' ὁ περὶ τῶν ἐν τῷ σταδίῳ κινουμένων ἐξ ἐναντίας ἴσων ὄγκων παρ' ἴσους, τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ' ἀπὸ μέσου, ἴσῳ τάχει, ἐν ᾧ συμβαίνειν οἴεται ἴσον εἶναι χρόνον τῷ διπλασίῳ τὸν ἥμισυν. ἔστι δ' ὁ παραλογισμὸς ἐν τῷ τὸ μὲν παρὰ κινούμενον τὸ δὲ παρ' ἡρεμοῦν τὸ ἴσον μέγεθος ἀξιούν τῷ ἴσῳ τάχει τὸν ἴσον φέρεσθαι χρόνον· τοῦτο δ' ἐστὶ ψεῦδος. οἷον ἔστωσαν οἱ ἐστῶτες ἴσοι ὄγκοι ἐφ' ὧν τὰ AA , οἱ δ' ἐφ' ὧν τὰ BB ἀρχόμενοι ἀπὸ τοῦ μέσου [τῶν A], ἴσοι τὸν ἀριθμὸν τούτοις ὄντες καὶ τὸ μέγεθος, οἱ δ' ἐφ' ὧν τὰ $ΓΓ$ ἀπὸ τοῦ ἐσχάτου, ἴσοι τὸν ἀριθμὸν ὄντες τούτοις καὶ τὸ μέγεθος, καὶ ἰσοταχεῖς τοῖς B . συμβαίνει δὴ τὸ πρῶτον B ἅμα ἐπὶ τῷ ἐσχάτῳ εἶναι καὶ τὸ πρῶτον $Γ$, παρ' ἄλληλα κινουμένων. συμβαίνει δὲ τὸ $Γ$ παρὰ πάντα τὰ B διεξεληλυθέναι, τὸ δὲ B παρὰ τὰ ἡμίση ὥστε ἥμισυν εἶναι τὸν χρόνον· ἴσον γὰρ ἐκάτερόν ἐστι παρ' ἕκαστον. ἅμα δὲ συμβαίνει τὸ πρῶτον B παρὰ πάντα τὰ $Γ$ παρεληλυθέναι· ἅμα γὰρ ἔσται τὸ πρῶτον $Γ$ καὶ τὸ πρῶτον B ἐπὶ τοῖς ἐναντίοις ἐσχάτοις, [ἴσον χρόνον παρ' ἕκαστον γιγνόμενον τῶν B ὅσον περ τῶν A , ὡς φησι,] διὰ τὸ ἀμφοτέρα ἴσον χρόνον παρὰ τὰ A γίγνεσθαι ὁ μὲν οὖν λόγος οὕτός ἐστιν, συμβαίνει δὲ παρὰ τὸ εἰρημένον ψεῦδος.

Unfortunately, the word “bodies” (ὄγκοι) appearing in the text of the *Stadium* is rather vague. In general the “ὄγκος” means “*bulk*,” “*size*,” “*mass*” of a body and therefore also “*body*” or “*human body*,” then, the “ὄγκοι” in plural means “*bodies*,” “*material substances*,” and sometimes “*particles*” or “*molecules*” in a special meaning.¹²⁵⁾ In any way it is not always clear what Zeno had

ZENO'S FOUR PARADOXES AGAINST MOTION

in mind by the word “ὄγκοι”. Some scholars like Tannery, Burnet, and Cornford advanced a hypothesis that the “ὄγκοι” in the *Stadium* refers after all to something like the Pythagorean “point-unit-atom.”¹²⁶⁾ In contradistinction to this however R. K. Gaye suggested that the “ὄγκοι” in question is due to Zeno who intended to criticize the “Empedoclean indivisible particles”¹²⁷⁾

These conceptions about “Pythagorean “point-unit-atom” or “Empedoclean indivisible particles” may be somehow evaluated; but, what relevance can those minute entities like indivisible “atoms” have to the “stadium”, where ordinarily some gymnastic contests like running-race among people take place? Why is it possible that something like the Pythagorean point-units or Empedoclean particles enter into the race-course? Can such a supposition of particles’ play in a stadium arouse people’s interest or sensation? Is it not out of place such conceptions? Is the title *Stadium* legitimate? Is it not due to Aristotelian misrepresentation?¹²⁸⁾ Or is it surely due to Zeno himself who gave a lecture, like a resourceful teacher of mathematics or physics, who put down a rectangular figure on the sand (or the board) with a piece of wood (or a chalk) and spoke as follows?

‘Let it be a “race-course” and let it be that these graphics (as represented ὄγκοι) stand for “xs” which are imagined vividly as “ys” on the race-course.

I do not know which the case was. In any case, there is nothing we can do about this issue. We are eventually in Aristotle’s hand,¹²⁹⁾ as though we were a swash-buckling monkey in the Great Buddha’s hand.¹³⁰⁾

However, Simplicius’ words may be suggestive about the matter. He said

that Eudemus the author of *The History of Geometry* called the “ὄγκοι” in this context “cubes” (κύβοι).¹³¹⁾ And Lee gave a reasonable comment to Simplicius’ saying:

‘I cannot suggest any reason why Eudemus called the bodies cubes, except that cubes are, as a matter of fact, very convenient for purposes of illustration. For the moving ὄγκοι are of course three-dimensional and if we take them to be cubes we can represent them two-dimensionally in our diagram easily enough by squares, the square being the side of the cube ... fits very well the “cinematographic” motion...’¹³²⁾

Useful though Lee’s comment is, it does not always scratch where we itch. He mentioned only to the reason for the graphic-notational convenience. What we want to know is the reason why Zeno adopted the term “ὄγκοι”, which should have been represented with a pictorial image (let it be *ys*) and which should have stood for the pluralist’s material entities (let it be *xs*).

Leaving aside the last question a while, about the latter question, I believe, there is a not so irrelevant answer. Let me remind the circumstances where Parmenides’ opponents ridiculed his doctrine of “what-is” (τὸ εἶν). It is a noteworthy fact that Parmenides likened his “what is” to “the *bulk* of a sphere” (σφαίρης ... ὄγκῳ fr. 8. 43) which is always “one” (ἓν),¹³³⁾ “indivisible” (οὐδὲ διαίρετόν)¹³⁴⁾ and “changeless” (ἀκίνητον).¹³⁵⁾ Having this case in mind firmly you should pay attention to another fact that Zeno intended to retort the opponents of his teacher by making clear the case that their own hypothesis that “plurality is” suffers still more absurd consequences than Parmenides’ “one”.¹³⁶⁾ Then, you will have a glimmering of a fact that Zeno may

ZENO'S FOUR PARADOXES AGAINST MOTION

attacked his adversaries' "ὄγκοι" (in plural) in order to defend Parmenides' "ὄγκος" (in singular). It is proverbially said that 'an eye for an eye.' Adhering to the *lex talionis* Zeno may have intended to chase his adversaries into self-contradiction.

However, Parmenides' τὸ εἶόν was likened to "the *bulk* of a sphere" (σφαιρῆς ... ὄγκῳ), not to the "cube" (κύβου). What is wrong with spherical bodies like golf-balls or foot-balls? R. Ferber thought there is nothing wrong with them and insisted as follows:

'Der Ausdruck ὄγκος wird von Zenon wieder aufgenommen, und vielleicht waren auch ὄγκοι im Stadium εὐκύκλοι, wohlgerundet, um das Wettrennen zu machen.'¹³⁷⁾

But, as you know well, in reality, the spherical body like a foot ball or a golf ball is unsuitable for the context of the *Stadium*. Ponder on again Zeno's stance. He attacked the pluralists who ridiculed Parmenides' τὸ εἶόν, which was expressed as

'τῷ ξυνεχὲς πᾶν ἐστίν· ἐὸν γὰρ ἐόντι πελάζει.'

(Thus, it is all continuous, for what is draws near to what is.)¹³⁸⁾

Zeno's adversaries who attacked Parmenides retained in their own part some theoretical devices which were a match for Parmenidean one; namely a theoretical model to depict the gapless or possibly seamless world-order.

A ball, a voluminous object, is unsuitable for such a theoretical model. Imagine the dispersed billiard balls on the table. When two billiard balls stand

linearly in a row, each of them will (theoretically) have just a point in common, because each of them is a spherical body and the two spherical bodies (ὄγκοι) which are in adjacent relation have just a point in common.¹³⁹⁾ It is theoretically impossible to fill up Euclidean space with such voluminous spheres. Thus, it is obvious in advance that one who intends to construct the world-order by a number of indivisible spherical *bodies* will become immediately a prey of Parmenidean logic.¹⁴⁰⁾ Thus ones who could attack him must have equipped themselves with some more potent devices in order to avoid such a flaw.

Plato did not inform us who the pluralists in question were. But, irrespective of who was the inventor of such a device, the “cubes” which were connected closely with one another without any gap could be a potent theoretical model for the pluralistic world-view which was in rivalry with Parmenides’ τὸ ἐόν, provided that they should be regarded as the “ὄγκοι” which stand for the “indivisible” and “minimal” quanta.

So far I have argued for an aspect of ὄγκοι as a theoretical model. Now let me turn to the topic of the setting of the *Stadium*. Aristotle’s report of this paradox is lengthy and intricate. It is not only intricate but also obscure. Even the readings of the transmitted Greek texts themselves are so manifold that the various interpretations have been given. And yet almost all interpretations were rather banal and inarticulate ones. A part of the main cause of this disappointing fact is surely due to the vagueness of Aristotle’s words about the location of “ὄγκοι.” Here I quote again the opening part of Aristotle’s report with a tentative translation:

‘Τέταρτος δ’ ὁ περὶ τῶν ἐν τῷ σταδίῳ κινουμένων ἐξ ἐναντίας ἴσων ὄγκων

ZENO'S FOUR PARADOXES AGAINST MOTION

παρ' ἴσους, τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ' ἀπὸ μέσου, ἴσῳ τάχ-
ει, ἐν ᾧ συμβαίνειν οἴεται ἴσον εἶναι χρόνον τῷ διπλασίῳ τὸν ἥμισυν.'

(‘The fourth is the one about the equal bodies which move past each other in a stadium in opposite directions, the one from the goal of the stadium, the other from the middle-point with equal velocities. This, he thinks, involves the conclusion that half a given time is equal to its double.’)

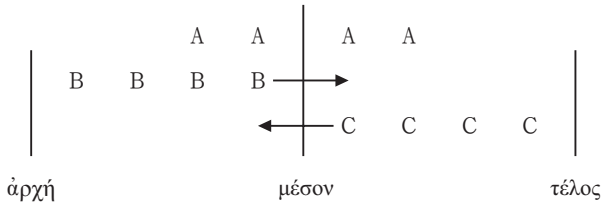
All the difficulties are derived from the vagueness of interrelationship between “τέλος” (the end) and “μέσον” (the middle) in the above quoted sentence ‘τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ' ἀπὸ μέσου’ (literally, ‘some from the end of the stadium on the one hand, and some from the middle on the other hand’). The standard rendering since Simplicius was one in the following figure 4. But, there is a flaw in this figure.

Lee points out the flaw as follows: ‘The *Cs* do start from the end (τέλος) and the *Bs* from the middle point (μέσον); but whereas it is the *last* or hindmost of the *Cs* which is on the finishing line (τέλος), it is the first or leading *B* which is on the middle-line (μέσον). And therefore, ‘if we describe the *Cs* as starting from the τέλος and the *Bs* from the μέσον, our description is asymmetrical as between *Cs* and *Bs*.’¹⁴¹⁾

However, Lee himself attributes this flaw to Aristotle’s lecture-style who ‘made use of a diagram, to which he was continually pointing,’ and says that

‘This passage is therefore certainly intended to be understood in conjunction with a diagram. If then we suppose him to have used a diagram similar to that given as fig. 2, it does not seem inconceivable that he should

Figure 6 (Lee's Fig. 2)



have spoken of the Cs starting to move from τὸ τέλος, and the Bs from τὸ μέσον, even though the reference is asymmetrical. For his meaning could be made quite clear by pointing; and if he pointed *first* to the τέλος, the μέσον would of course *come next* as he moved his hand or pointer from right to left, and so he might easily describe the position of the Cs by reference to the τέλος, and that of the Bs by reference to the μέσον.¹⁴²⁾

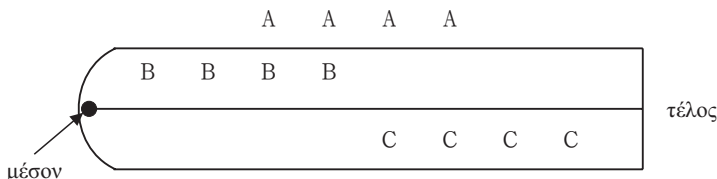
Saying so he rejected an alternative interpretation presented by W. D. Ross who 'is dissatisfied with this traditional rendering' and who 'takes μέσον in the sense of the turning point in the double course.' (See below the figure 7 = Lee's Fig. 3)

The only difficulty in accepting Ross' proposal which Lee detected was that 'there is no authority for giving μέσον this sense.' Thus Lee continues to say as follows:

'I can find no instance of it being so used in any ancient author, and in view of the archaeological evidence it seems to be extremely doubtful whether Aristotle could so used it.'¹⁴³⁾

ZENO'S FOUR PARADOXES AGAINST MOTION

Figure 7



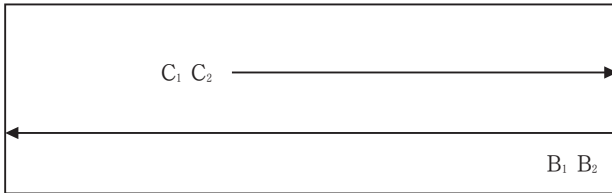
Thus in a similar way a number of scholars tried to reconstruct the setting of the *Stadium* without getting to a conclusive solution. And some of them expressed their scepticism to Aristotle's report and the reliability of traditional text.

R. Ferber's interpretation which has been regarded among some scholars as one of the excellent achievement in the recent study of Zeno's paradoxes too can be regarded as a result of such a scepticism against Aristotle and his text. The outline of his interpretation can be shown in the following figure:¹⁴⁴⁾

According to Ferber's interpretation, the *Stadium* is the reversed version of the *Achilles*. In the *Achilles*, he insists, the tortoise and Achilles make trip in the same direction, whereas in the *Stadium* two groups of ὄγκοι moves in the opposite direction. One of the groups, the swifter one, has reached the goal just now and is about to return back to the starting point, when the slower one is barely at the middle point of the stadium.

But, I think, this is a very dubious and arbitrary interpretation. It is wholly unclear why two groups of ὄγκοι, the one of which is originally swifter than the other, should in the next stage begin to move at the same speed in the opposite direction. Originally in Aristotle's report there is no mention to the two moving bodies at different speeds. They are just an arbitrary fabrication by

Figure 8



Ferber who sought his own interests to identify the interrelationship between μέσον and the τέλος. In addition to this, in Ferber's diagram we cannot exactly specify where the leading part of B₁B₂ is, whereas it should become an essential and urgent issue which must not be left as it is, if the B₁B₂ were really to be ὄγκοι (voluminous objects) and should pass by the other voluminous objects C₁C₂.

To tell the truth, the ὄγκοι in plural have no significant role within the framework of Ferber's interpretation. They are, according to him, irrelevant to the essentials of the *Stadium*. In addition to this, it matters little whether the two groups of ὄγκοι move towards the opposite direction or not. In any way, they are, according to Ferber, merely the fabrications produced by Aristotle. Thus Ferber says:

‘Dabei scheint auch noch Abschnitt (a) [*Phys.*, Z 9. 239b34–240a1; Yamakawa's note] von Aristoteles verändert worden zu sein. Denn das Zenonische Argument involviert natürlich nicht notwendig, daß sich die beiden Massengruppen aus *entgegengesetzter* Richtung bewegen. Sie können das auch aus *gleicher* tun. Es involviert nicht einmal, daß sich zwei Massen-*gruppen* bewegen, vielmehr *genügen* auch zwei *einzelne* Massen.

ZENO'S FOUR PARADOXES AGAINST MOTION

An der Substanz des Zenonischen Arguments ändern *diese* aristotelischen Umdeutungen freilich nichts.¹⁴⁵⁾

Is this interpretation acceptable? No, this interpretation seems among all to ignore Ferber's own maximum: 'Philosophiehistorie ohne systematische Philosophie ist blind, systematische Philosophie ohne Philosophiehistorie ist leer.'¹⁴⁶⁾ We must look for a more natural and historically sensitive interpretation. Does such an interpretation still remain? It is at hand, I believe, in a variant of Ross' view, which Ferber superficially dismissed as follows:

'Im bewußtsein dieser Schwierigkeiten meint W. D. Ross, daß mit der Mitte des Stadiums der Wendepunkt eines (dann elliptischen oder kreisförmigen) Arealis gemeint sei. Doch muß er zugeben, daß auch diese Interpretation problematisch ist: Denn der gewöhnliche Name für den Wendepunkt eines solchen Stadiums (δίαιλος) lautet der "καμπτήρες". Er muß weiter zugeben, daß der δίαιλος nur eines von vielen Rennen war, und ein Ende der Rennbahn, wenn nur eine Runde gelaufen wurde, nicht Mitte genannt werden konnte. Er akzeptiert denn auch diese Interpretation nur faute de mieux.'¹⁴⁷⁾

But, this seems to be a myopic view. Open the Book XXIII of the *Iliad* and read thoroughly the paragraph where the chariot race in honour of Patroclus takes place.

At lines 304 ff. we witness Nestor's fatherly figure who gives his son Antilochus a good advice as follows:

- (1) ‘You are skilful at *turning the chariot round the doubling-post* (περὶ τέρμαθ’ ἔλισσόμεν 309), but the horses themselves are very slow, and it is this that will, I fear, mar your chances.’
- (2) ‘There is a *stump* (σῆμα 326) of a dead tree-oak or pine as it may be some six feet above the ground,’ ...
- (3) ‘it stands *at the turning point of the road* (ἐν ξυνοχῆσιν ὁδοῦ 330) and’
- (4) ‘*there is a smooth chariot-course all round it* (λείος δ’ ἵππόδρομος ἀμφίς 330).’
- (5) ‘It may have been used as *the turning-post* [which driving up the right side of the course, turned round it, and returned by the left side] (νύσσα 332) in days gone by.’
- (6) ‘Now, however, it has been fixed on by Achilles as *the mark round which the chariots shall turn*’ (σῆμα ... τέρματ’ 331-333);
- (7) ‘hug it as close as you can, but as you stand in your chariot lean over a little *to the left* (ἐπ’ ἀριστερά 336); urge on your *right-hand horse* (τὸν δεξιὸν ἵππον 336) with voice and lash, and give him a loose rein,’
- (8) ‘but let *the left-hand horse* (ἵππος ἀριστερὸς 338) keep so close in, that the nave of your wheel shall almost graze
- (9) *the turning-post*. (νύσση 338)’

And when the five charioteers, Antilochus, Eumelus, Menelaus, Meriones and Diomed, took their places in line, Homer continues to tell the story:

- (10) ‘*Achilles showed them the doubling post round which they were to turn,*

ZENO'S FOUR PARADOXES AGAINST MOTION

some way off upon the plain' (σήμνε δὲ τέρματ' Ἀχιλλεύς τηλόθεν ἐν λείῳ πεδίῳ 358-359).

- (11) 'And there Achilles stationed his father's follower Phoenix as umpire, to note the running and report truly.' (παρὰ δὲ σκοπὸν εἶσεν ἀντίθεον Φοίνικα, ὀπάονα πατρὸς ἑοῖο, ὡς μεμνέωτο δρόμου καὶ ἀληθείην ἀποείποι. 359-361)
- (12) 'At the same instant all of them lashed their horses, struck them with the reins, and shouted at them with all their might. They flew full speed over the plain *away from the ships* (νόσφι νεῶν 365).'
- (13) 'It was when they were doing the last part of the course on their way back *towards the grey sea* (ἐφ' ἄλῳς πολιῆς 374) that their pace was strained to the utmost and it was seen what each could do.'

'Meanwhile the Achaeans from their seats were watching how the horses went, as they scoured the plain amid clouds of their own dust. ... He [Idomeneus] stood up and said among the Argives: "My friends, princes and counsellors of the Argives, can you see the running as well as I can? ... Those that led off at the start must have been disabled out on the plain.' And he says:

- (14) I saw them at first *making their way round the doubling post*' (περὶ τέρμα βαλούσας 462). ...
- (15) Perhaps the reins fell from the driver's hand so that *he lost command of his horses at the doubling post, and could not turn it.* (οὐδὲ δυνάσθη εὖ σχεθέειν περὶ τέρμα, καὶ οὐκ ἐτύχησεν ἐλίξας 465-466)."

In the above long quotation you could immediately distinguish the conditions which correspond one by one to the *Stadium*. In order to confirm the case, though it may be regarded as a detour, let me refer to Ross' claim in his commentary to the *Stadium*. He said [italics by Yamakawa]:

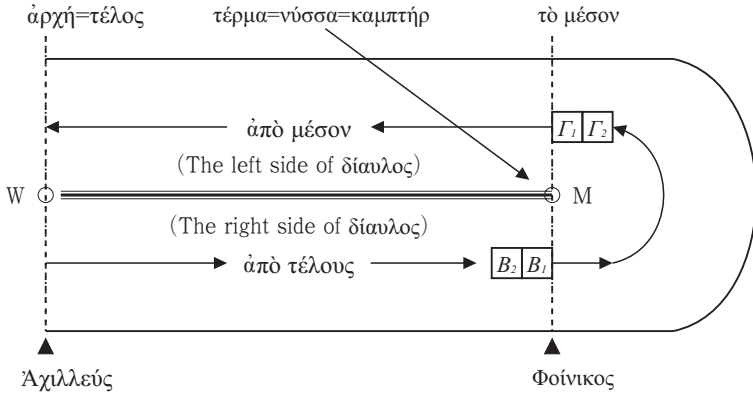
The solution lies in the supposition that τὸ μέσον means the *middle of the complete course along the stadium and back again*; in other words *the turning-point*. ἀπὸ μέσον, ἀπὸ τοῦ τέλους then refer to *direction, not to position*, and the particular position of the A's relative to the length of the stadium is immaterial, as in view of the general nature of the argument it ought to be. ... It must be admitted that τὸ μέσον is apparently not used elsewhere of the turning-point of the δίαυλος, the ordinary name for which is καμπτήρες. It must be admitted also that the δίαυλος was only one out of many races that were run in the stadium, and that *when only one length was run, one end of the race-course could not be called the μέσον*. But if Aristotle had in mind any race in which an even number of lengths was run, he might naturally refer to the καμπτήρες as the μέσον; and a *gesture by him as he drew the figure would make plain what he meant by the μέσον*. The unnaturalness of the alternative interpretation of the passage seems to me to outweigh the lack of other evidence for our interpretation of τὸ μέσον.^{*148)}

Ross referred to 'the lack of other evidence.' But, I believe that the above mentioned setting of the chariot-race in the *Iliad* XXIII affords a strong evidence which supports fully Ross' conjecture that τὸ μέσον means the turning-point. The following is the reason for this.

ZENO'S FOUR PARADOXES AGAINST MOTION

1. The νόσσα at line 332 & 338 is synonymous with καμπτήρ; it is the turning-post which stands at the middle-point [M] of the entire course [(5) (9)].
2. The νόσσα is called also τέρμα or σῆμα [(1) (2) (6) (10) (14) (15)] which stands at the turning point [M] of the entire race-course [(3) (4)].
3. Thus the chariot-race has three steps:
 - (i) Driving up the right side of the double course (δίαυλος) and arrive at the middle point (turning-point) [(7) (8) (12) (13)];
 - (ii) turning round the doubling-post towards the opposite direction [(7) (8) (12) (13)];
 - (iii) returning back to the winning post by the left side course of the double course (δίαυλος) [(13)].
4. There are two persons who act as umpires; the one is Achilles at the winning post [W];¹⁴⁹⁾ the other is Phoenix at the turning-point whose role is *to note the running and report truly* [(10) (11)].
5. It is obvious that the chariot-race in the Book XXIII of the *Iliad* is a kind of διάυλος because the chariot runs 'full speed over the plain away from the ships and returns back 'towards the grey sea' [(12) (13)]. It is a race on the double course (δίαυλος) in which the chariot runs in the first place to the furthest point (=the middle point) of the entire race-course, then turning the νόσσα [=καμπτήρ], and lastly runs back to the starting point by the right side course.
6. Therefore, the νόσσα [=καμπτήρ] is the middle-point of the entire race-course.
7. The expressions "ἀπὸ μέσου" and "ἀπὸ τοῦ τέλους" in this context

Figure 9

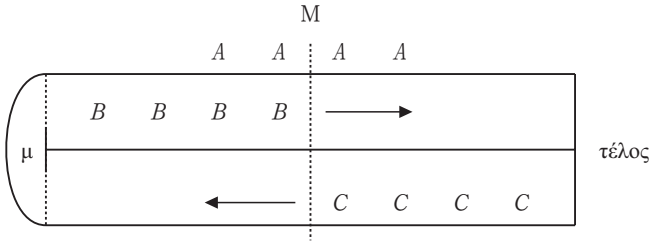


naturally refer, as Ross said legitimately, to direction, not to position. Thus, viewed from this point of view, ἀρχή and τέλος may be regarded as equivalent one another and replaceable.

8. Aristotle's words that 'the equal bodies which move past each other in a stadium in opposite directions, the one from the goal of the stadium, the other from the middle-point with equal velocities' in conjunction with the above mentioned conditions necessitate us to suppose the opposite movements of the two rows of bodies which pass by each other at the turning-point (= the middle-point) in question. There is no reason to believe that these two rows of equal bodies should pass by each other in the middle-point in the sense which Lee *et al.* understood it.
9. Concerning the details of the setting of two rows passing by each other at the turning point (M), it may be appropriate to imagine as follows:
 - (1) The one row (ὄγκοι) which consists of Γ_1 and Γ_2 corresponds to

ZENO'S FOUR PARADOXES AGAINST MOTION

Figure 10



a couple of Eumelus and Diomed and the other row of B_1 and B_2 to that of Menelaus and Antilochus.

10. It is noticeable that each of the couples consists of a set of two rivals who respectively develop a desperate struggle between them. See for example the case between Eumelus and Diomed: ‘The horses of the descendant of Pheres now took the lead, and close behind (μάλ’ ἐγγύς) them came the Trojan stallions of Diomed. They seemed as if about to mount Eumelus’ chariot, and he could feel their warm breath on his back and on his broad shoulders, for their heads were close to him as they flew over the course.’¹⁵⁰⁾ The speed of Diomed’s chariot suffers nothing by comparison with one of Eumelus’. The case between Menalaus and Antilochus too is much the same.¹⁵¹⁾
11. There is no gap between two “bodies” (ὄγκοι), the situation of which can be suitably represented by two “cubes” which are closely adjacent to one another.

It is noteworthy that the figure 9 involves some fundamentally important changes from Ross’ original interpretation (See Figure 10).

Ross' greatest weak point consists in his supposition of *implicit* double use of the term "the middle." The "middle" means on the one hand a turning-point (μ); but on the other hand the middle-point (M) of the stadium. As a consequence of this double use of the "middle" it became necessary for Ross to make another unnatural supposition, i.e., to guess that the leading *C* too starts to move at the middle point M, being urged by a fact that the leading *B* must start 'ἀπὸ μέσου' as the above figure 10 shows.

In contrast with this weakness, my interpretation has not only a historical support in Homer, but also is consistent with the structure of the Greek stadium. My representation of the *Stadium* satisfies the necessary conditions which Greek traditional stadium must fulfil. In order to verify the case I quote Lee's explanation about Greek stadium. He says as follows:

'In shape it was rectangular. Starting and finishing line were marked by a line of stone slabs, in which are sockets for posts. These posts seem to have been called *καμπτήρες* (note the plural), and were temporary, having to be renewed each time the games were held. They seem to have been used both in the stadium-race and in the *δίαιλος*: in the stadium as objectives, in the *δίαιλος* as an actual turning posts. In the long races all these posts except the central one were removed, and the runners all turned round it. *There was in fact a single turning post only in the long races.*^{*152)}

In the above figure 9 there are no inconsistent items with Lee's above testimony.

ZENO'S FOUR PARADOXES AGAINST MOTION

So far we have successfully excavated the first layer of Zeno's four paradoxes which was covered by a large quantity of surface soil and brought many "invisible" aspects to light. Now, we are prepared to dig up afresh the second layer of Zeno's arguments against motion.

Notes

- 1) Maurice Merleau-Ponty, *Le Visible et l'Invisible suivi de notes de travail*, Éditions Gallimard, 1964, p. 300.
- 2) In order to convey my intention of this book I composed the following poems all of which represent a rather straightforward visual intimation of the "invisible" meanings implied by Zeno's "paradoxes" which may be metaphorically summarized under the title of "Achilles and his Shadow." I hope the readers will through going reading of this book appreciate the metaphorical meanings suggested previously in these poems.
- 3) This is an adaptation of Homer, *The Iliad*, XXII. 21-23 making use of the last scene of *Hashire Meros!* (*Hurry now Meros!*) by Japanese novelist Dazai Osamu.
- 4) This is an adaptation of Zeno's *Achilles* making use of Peter Schlemiel's *The Shadowless Man*.
- 5) This is also another adaptation of *Achilles* making use of Chinese Taoist philosopher Chuan-tzu's work 'A dialogue between the outline and the shadow' (*The Inner chapter 3*). Cf. *The Book of Chuang-tzu*, A new, complete translation of *the classic Taoist text*, translated by Martin Palmer with Elizabeth Breuilly, Arkana Penguin Books, 1996, p. 20.
- 6) This is an adaptation of Zeno's *Arrow* making use of *The Iliad*, XXII. 188-292.
- 7) Maurice Merleau-Ponty, *Le Visible et l'Invisible suivi de notes de travail*, Éditions Gallimard, 1964, p. 269.
- 8) When the Persians captured the Ionian cities, according to Herodotus' sympathetic report (I, 164-5), the Phocaeans decided to abandon their native city and sailed to the westward world. They explored the Adriatic Sea and the far west to find out the prominent colonies including Messalia and Tartessos in Spain. The Phocaeans who settled at Alalia were forced to Corsica after five years under the

attacks of the allied Carthaginian and Etruscan and finally made a permanent home at Velia (Elea). See L. H. Jeffery, *Archaic Greece*, The city-states c. 700-500 B.C., Methuen & Co. Ltd, London, 1976, p. 228; see also C. J. Emlyn-Jones, *The Ionians and Hellenism*, A Study of the cultural achievement of the early Greek inhabitants of Asia Minor, Routledge & Kegan Paul, 1980, pp. 22ff.

- 9) According to *DL* 28 Elea, Zeno's native place, is a Phocaean colony, 'once known as Hyele,' 'a city of moderate size, skilled in nothing but to rear brave men.'
- 10) Plato in his dialogue *Parmenides* 127a-c describes a meeting between Parmenides, Zeno, and Socrates: 'Zeno and Parmenides once came to Athens for the Great Panathenaea. Parmenides was then well advanced in years, about sixty-five, quite grey-haired, and of distinguished appearance, and Zeno was nearly forty, handsome and tall. It was said he had been Parmenides' favorite. He said that they stayed at Pythodorus' house in Ceramicus, outside the city walls, and Socrates came there with a number of others, eager to hear a reading of Zeno's treatise, which Zeno had brought to Athens for the first time. Socrates at the time was very young.' Following to this description we may say that Zeno was born about 490 B.C.
- 11) Cf. Plato, *Phaedrus*, 261d: 'Do we not know that the Eleatic Palamedes has such an art of speaking that the same things appear to his hearers to be alike and unlike, one and many, stationary and in motion?' (translation by Cornford) Though some scholar regards Zeno's nick-name "Eleatic Palamedes" given by Plato as a contemptuous epithet, in reality it is not the case. Contra Cornford's understanding of the art of disputation *ἀντιλογική* of Zeno the "Palamedes," R. E. Allen aptly says: Paramedes was the Inventor, a culture-hero like Prometheus, who gave to Greek the adjective *παραμήδειος*, "ingenious." He is one of the people Socrates hoped to meet in the afterlife (*Apology* 41b), and a symbol of wisdom as opposed to political power in *Epistle* II (311b).' See R. E. Allen, *Plato's Parmenides*, Translated with Comment, Yale University Press, 1997, p. 77.
- 12) Cf. According to Plutarch's *Life of Pericles*, ch. 4.3 it is said that 'Pericles also a student of Zeno the Eleatic, who discouraged on physics, like Parmenides, and

ZENO'S FOUR PARADOXES AGAINST MOTION

who perfected a kind of skill in examining opponents in argument that brought them to a state of *aporia* through opposed arguments; so Timon of Phileus expressed it, when he spoke of the great power, that failed not in its effect, of Zeno with the two-edged tongue, the man who laid hold upon all things.' Cf. also Plato, *Alcibiades I*, 119a; see also W. K. C. Guthrie, *A History of Greek Philosophy*, Vol. II, Cambridge University Press, 1969, pp. 80-81.

13) Cf. *DL IX. 28*.

14) Cf. *DL IX. 25*: 'Aristotle says that Zeno was the inventor of dialectic, as Empedocles was of rhetoric.' Cf. also *DL. VIII. 57*: 'Ἀριστοτέλης δ' ἐν τῷ Σοφιστῇ [fr. 65] φησι πρῶτον Ἐμπεδοκλέα ῥητορικὴν εὐρεῖν, Ζήνωνα δὲ διαλεκτικὴν. Cf. also Sext., *adv. math.* VII. 6: Παρμενίδης δὲ οὐκ ἂν δόξαι τῆς διαλεκτικῆς ἀπειρώς ἔχειν, ἐπεὶ πάλιν Ἀριστοτέλης τὸν γνώριμον αὐτοῦ Ζήνωνα διαλεκτικῆς ἀρχηγὸν ὑπείληφεν.

15) See 'Appendix A: Zeno's Writings.'

16) Cf. *DL IX. 26*.

17) Diogenes Laertius (Cf. *DL IX. 26-27*) says: 'He plotted to overthrow Nearchus the tyrant (or according to others, Diomedon) but was arrested: so Heraclides in his epitome of Satyrus. On that occasion he was cross-examined as to his accomplices and about the arms which was conveying to Lipara; he denounced all the tyrant's own friends, wishing to make him destitute of supporters. Then, saying that he had something to tell him about certain people in his private ear, he laid hold of it with his teeth and did not let go until stabbed to death, meeting the same fate as Aristogiton the tyrannicide. Demetrius in his work on *Men of the Same Name* says that he bit off, not the ear, but the nose. According to Antisthenes in his *Successions of Philosophers*, after informing against the tyrant's friends, he was asked by the tyrant whether there was anyone else in the plot; whereupon he replied, "Yes, you, the curse of the city!"; and to the bystanders he said, "I marvel at your cowardice, that, for fear of any of those things which I am now enduring, you should be the tyrant's slaves." And at last he bit off his tongue and spat it at him; and his fellow-citizens were so worked upon that they forthwith stoned the tyrant to death. In this version of the

- story most authors nearly agree, but Hermippus says he was cast into a mortar and beaten to death.’
- 18) Plato, *Parmenides*, 128a, translation by Cornford. We should bear a fact in mind that an English translation “there is a One” is only an alternative between various expressions which may convey the meanings respond to the original Greek expression “εἰ ἓν ἐστι.” See R. E. Allen’s notes 5 & 6 which refers to the expression “εἰ πολλά ἐστιν” in his *Plato’s Parmenides*, Translated with Comment, Revised Edition, Yale University Press, 1997, pp. 5-6.
- 19) This is a reason why there was some talk that he was a ‘double-tongued’ person.
- 20) See for example the description of the contemporary usage of the “paradox” in *Longman Lexicon of Contemporary English*, Longman Group Ltd., 1981, p. 328.
- 21) They say that Philetas of Cos was tormented by the Liar Paradox and committed suicide. See note 22 R. M. Sainsbury, *Paradoxes*, Cambridge University Press, Second edition, 1995, p. 1.
- 22) See Sainsbury’s definition of the “paradox.” Cf. R. M. Sainsbury, *Paradoxes*, Cambridge University Press, Second edition, 1995, p. 1. Sainsbury says: ‘Appearances have to deceive, since the acceptable cannot lead by acceptable steps to the unacceptable. So, generally, we have a choice: either the conclusion is not really unacceptable, or else the starting point, or the reasoning, has some non-obvious flaw.’
- 23) Cf. *DL*, IX, 25. Timon called Zeno and Melissus “double-tongued:” ἀμφοτερογλώσσου τε μέγα σθένος οὐκ ἀλαπαδὸν / Ζήνωνος πάντων ἐπιλήπτορος, ἡδὲ Μέλισσον./πολλῶν φαντασμῶν ἐπάνω, παύρων γε μὲν ἦσσω. Cf. also Proclus, *Commentary on Plato’s Parmenides*, Translated by Glenn R. Morrow and John M. Dillon, Princeton University Press, 1987, 631.36-632.23.
- 24) Cf. Maurice Merleau-Ponty, *Le Visible et l’Invisible suivi de notes de travail*, Éditions Gallimard, 1964, p. 269
- 25) τέτταρες δ’ εἰσὶ λόγοι περὶ κινήσεως Ζήνωνος οἱ παρέχοντες τὰς δυσκολίας τοῖς λύουσι.
- 26) See Aristotle, *Phys. Z.* 2, 233a21.

ZENO'S FOUR PARADOXES AGAINST MOTION

- 27) The text of the *Dichotomy*: πρώτος μὲν ὁ περὶ τοῦ μὴ κινεῖσθαι διὰ τὸ πρότερον εἰς τὸ ἥμισυ δεῖν ἀφικέσθαι τὸ φερόμενον ἢ πρὸς τὸ τέλος, περὶ οὗ διείλομεν ἐν τοῖς πρότερον λόγοις. διὸ καὶ ὁ Ζήνωνος λόγος ψευδὸς λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἄψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μήκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἦτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἄψασθαι ἐν πεπερασμένῳ χρόνῳ τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διεναί τὸ ἄπειρον καὶ ἄπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις.
- 28) Cf. *Topics*, 160b6-9: 'Although even that is not enough; for we often hear arguments that are contrary to common opinions, whose solution is yet difficult, e.g. the argument of Zeno that it is impossible to move or to traverse the stadium;' See also Aristotle, *Phys.* Θ8 263a5; Simplicius, 1013. 4 ad 239b10; 947. 5 ad 233a21; 1289. 5 ad 263a5.
- 29) Bekker reads βραδύτερον. But I take βραδύτατον following Cornford, Lee, and Ross.
- 30) The text of the *Achilles*: δεύτερος δὲ ὁ καλούμενος Ἀχιλλεύς. ἔστι δ' οὗτος ὅτι τὸ βραδύτατον οὐδέποτε καταληφθήσεται θεόν ὑπὸ τοῦ ταχίστου· ἔμπροσθεν γὰρ ἀναγκαῖον ἐλθεῖν τὸ διώκον ὅθεν ὥρμησε τὸ φεῦγον, ὥστ' αἰεὶ τι πρόχειν ἀναγκαῖον τὸ βραδύτατον.
- 31) Cf. A. C. Graham, *Chuang-tzu*, The Inner Chapters, Unwin Paperbacks, 1986, p. 56.
- 32) Following Bekker's reading βραδύτερον at 239b21 Lee retains it, whereas he emended other βραδύτερον of Bekker's text to βραδύτερον. However, he mis-translated this βραδύτερον to 'the slowest runner.'
- 33) The text of the *Achilles* (Second part): ἔστι δὲ καὶ οὗτος ὁ αὐτὸς τῷ διχοτομεῖν, διαφέρει δὲ ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβάνεσθαι τὸ βραδύτερον συμβέβηκεν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταῦτὸ τῇ διχοτομία—ἐν ἀμφοτέροις γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρασ διαιρουμένου πως τοῦ μεγέθους· ἀλλὰ προσκεῖται ἐν τούτῳ ὅτι οὐδὲ τὸ

- τάχιστον τετραγωδημένον ἐν τῷ διώκειν τὸ βραδύτατον—ὥστ' ἀνάγκη καὶ τὴν λύσιν εἶναι τὴν αὐτήν.
- 34) The text of the *Arrow I*: Ζήνων δὲ παραλογίζεται· εἰ γὰρ αἰεὶ, φησίν, ἥρεμει πᾶν ἢ κινεῖται, <οὐδὲν δὲ κινεῖται> ὅταν ἦ κατὰ τὸ ἴσον, ἔστι δ' αἰεὶ τὸ φερόμενον ἐν τῷ νῦν κατὰ τὸ ἴσον, ἀκίνητον τὴν φερομένην οἰστόν. τοῦτο δ' ἔσται ψεῦδος· οὐ γὰρ συγκεῖται ὁ χρόνος ἐκ των νῦν τῶν ἀδιαιρέτων, ὥσπερ οὐδ' ἄλλο μέγεθος οὐδέν.
- 35) The text of the *Arrow II*: τρίτος δ' ὁ νῦν ῥηθεῖς, ὅτι ἡ οἰστός φερομένη ἔστηκεν. Συμβαίνει δὲ παρὰ τὸ λαμβάνειν τὸν χρόνον συγκεῖσθαι ἐκ τῶν νῦν· μὴ διδομένου γὰρ τούτου οὐκ ἔσται ὁ συλλογισμός.
- 36) The fact may be indicative of a fact that Aristotle especially concerned about the *Arrow*.
- 37) Reading τοῦ μέσου τῶν *A*. EHI, Ross; Lee omit τῶν *A*.
- 38) Cf. Bekker, Lee: reading πάντα τὰ *B*;
- 39) Reeding after Lee: τὸ δὲ *B*.
- 40) After Ross and Lee, om. ἴσον χρόνον ...ὡς φησι as a gloss on ἴσον γὰρ ἕκατερον ὅν ἐστι παρ' ἕκαστον in l. 6.
- 41) The text of the *Stadium*: Τέταρτος δ' ὁ περὶ τῶν ἐν τῷ σταδίῳ κινουμένων ἐξ ἐναντίας ἴσων ὄγκων παρ' ἴσους, τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ' ἀπὸ μέσου, ἴσῳ τάχει, ἐν ᾧ συμβαίνειν οἴεται ἴσον εἶναι χρόνον τῷ διπλασίῳ τὸν ἡμισυν. ἔστι δ' ὁ παραλογισμὸς ἐν τῷ τὸ μὲν παρὰ κινούμενον τὸ δὲ παρ' ἥρεμοῦν τὸ ἴσον μέγεθος ἀξιούν τῷ ἴσῳ τάχει τὸν ἴσον φέρεσθαι χρόνον· τοῦτο δ' ἐστὶ ψεῦδος. οἷον ἔστωσαν οἱ ἐστῶτες ἴσοι ὄγκοι ἐφ' ὧν τὰ *AA*, οἱ δ' ἐφ' ὧν τὰ *BB* ἀρχόμενοι ἀπὸ τοῦ μέσου τῶν *A*, ἴσοι τὸν ἀριθμὸν τούτοις ὄντες καὶ τὸ μέγεθος, οἱ δ' ἐφ' ὧν τὰ *ΓΓ* ἀπὸ τοῦ ἐσχάτου, ἴσοι τὸν ἀριθμὸν ὄντες τούτοις καὶ τὸ μέγεθος, καὶ ἰσοταχεῖς τοῖς *B*. συμβαίνει δὴ τὸ πρῶτον *B* ἅμα ἐπὶ τῷ ἐσχάτῳ εἶναι καὶ τὸ πρῶτον *Γ*, παρ' ἄλληλα κινουμένων. συμβαίνει δὲ τὸ *Γ* παρὰ πάντα τὰ *B* διεξεληλυθέναι, τὸ δὲ *B* παρὰ τὰ ἡμίση ὥστε ἡμισυν εἶναι τὸν χρόνον. ἴσον γὰρ ἐκάτερόν ἐστι παρ' ἕκαστον. ἅμα δὲ συμβαίνει τὸ πρῶτον *B* παρὰ πάντα τὰ *Γ* παρεληλυθέναι ἅμα γὰρ ἔσται τὸ πρῶτον *Γ* καὶ τὸ πρῶτον *B* ἐπὶ τοῖς ἐναντίοις ἐσχάτοις [ἴσον χρόνον παρ' ἕκαστον γιγνόμενον τῶν *B* ὅσονπερ τῶν *A*, ὡς φησι,] διὰ τὸ ἀμφοτέ-

ZENO'S FOUR PARADOXES AGAINST MOTION

ρα ἴσον χρόνον παρὰ τὰ *A* γίνεσθαι. ὁ μὲν οὖν λόγος οὕτως ἐστίν, συμβαίνει δὲ παρὰ τὸ εἰρημένον ψεῦδος.

- 42) Cf. Simplicius' commentary: 'This being premised he goes on to suppose a stadium *DE*, and four bodies of equal size *AA*—or any number, provided it be even, so that the number of bodies ... has a half—which are stationary and are placed so as to occupy a central stretch of the stadium.' (translation by Lee) Thus it is necessary that the number of the bodies is even, because all the trains have to be divided into their halves. In this respect, as it will become clear, the *Stadium* is a version of the *Dichotomy*.
- 43) Cf. N. B. Booth, 'Zeno's Paradoxes,' *JHS*, 1957, II, p. 194.
- 44) Cf. G. S. Kirk, J. E. Raven and M. Schofield, *The Presocratic Philosophers*, Cambridge University Press, Second Edition, 1983, p. 276.
- 45) See Parmenides' fragment 7. 4.
- 46) Cf. Aristotle, *Rhetoric*, Vol. II, 1403a19–19; also Vol. I, 1362a20–21.
- 47) Cf. Aristotle, *Topics*, Vol. VIII, 163a29–33.
- 48) Cf. Simplicius, 1289. 5; Lee, p. 49.
- 49) Vlastos objects to Simplicius' view of the Dichotomy. But, I think, Simplicius is justified.
- 50) Cf. *Iliad*, XXII, 208.
- 51) Cf. Simplicius, Op. cit. ἐν παντὶ συνεχεῖ ἔσται ἄπειρα ἡμίση ...
- 52) Cf. Philoponus, 81. 7, Lee, p. 47.
- 53) W. D. Ross, *Aristotle's Physics*, A Revised Text with Introduction and Commentary, Oxford, (First edition 1936), 1960, p. 72.
- 54) Adolf Grunbaum, *Modern Science and Zeno's Paradoxes*, London, George Allen and Unwin Ltd, 1967., pp. 37–38.
- 55) Following Bekker's reading βραδύτερον at 239b21 Lee retains it, whereas he emended other βραδύτερονς of Bekker's text to βραδύτατον. However, he mistranslated this βραδύτερον to 'the slowest runner.'
- 56) The text of the *Achilles* (Second part): ἔστι δὲ καὶ οὗτος ὁ αὐτὸς τῷ διχοτομῆν, διαφέρει δὲ ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβάνεσθαι τὸ βραδύτερον συμβέβηκεν ἐκ τοῦ λόγου, γίνεταί δὲ παρὰ

ταῦτο τῆ διχοτομίας—ἐν ἀμφοτέροις γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρασ διαιρουμένου πως τοῦ μεγέθους· ἀλλὰ προσκεῖται ἐν τούτῳ ὅτι οὐδὲ τὸ τάχιστον τετραγῶδημένον ἐν τῷ διώκειν τὸ βραδύτατον—ὥστ’ ἀνάγκη καὶ τὴν λύσιν εἶναι τὴν αὐτήν.

- 57) See Simplicius, 1013. 31; Lee, p. 51: καὶ οὗτος ὁ λόγος ἐκ τῆς ἐπ’ ἄπειρον διαίρεσεως ἐπικεχρίρηται κατ’ ἄλλην διασκεύην. καὶ εἴη ἂν τοιοῦτος. εἰ ἔστι κινήσεις, τὸ βραδύτατον ὑπὸ τοῦ ταχίτου οὐδέποτε καταληφθήσεται. ἀλλὰ μὴν τοῦτο ἀδύνατον. οὐκ ἄρα ἔστι κινήσεις ... (1014.9) Ἀχιλλεὺς οὖν ὁ λόγος ἀπὸ τοῦ παραληφθέντος ἐν αὐτῷ Ἀχιλλεὺς ἐκλήθη, ὃν ἀδύνατόν φησιν ὁ λόγος τὴν χελώνην διώκοντα καταλαβεῖν.
- 58) Concerning the word-meaning “pompous”, “bombastic” of τραγωδεῖν, τραγωδία, Lee prompts us to see LS(J).
- 59) H.D.P. Lee, *Zeno of Elea*, A Text, with Translation and Notes, Amsterdam, Adolf M. Hakkert-Publisher, 1967, p. 77.
- 60) Cf. *The Iliad*, XXIII. 346.
- 61) W. D. Ross, *Op. cit.*, p. 71.
- 62) Cf. *The Iliad*, XXII. 139–140: ἦῤτε κίρκος ὄρεσφιν, ἐλαφρότατος πετεηνῶν,
- 63) Ibid: ῥηϊδίως οἴμησε μετὰ τρήρωνα πέλειαν ... In the other context, however, Homer likens Achilles to “a hound” (κύων) and Hector to “a fawn” (νέβρος). Cf. *The Iliad*, XXII. 189–190.
- 64) Cf. *The Iliad*, XXII. 199–201: ‘And as in a dream a man availeth not to pursue one that fleeth before him—the one availeth not to flee, nor the other to pursue—even so Achilles availed not to overtake Hector in his fleetness, neither Hector to escape.’ (ὡς δ’ ἐν ὄνειρῳ οὐ δύναται φεύγοντα διώκειν· οὔτ’ ἄρ’ ὁ τὸν δύναται ὑποφεύγειν οὔθ’ ὁ διώκειν/οὐ δύνατο μάρψαι πόσιν, οὐδ’ ὃς ἀλύξαι.) Translation by A. T. Murray (Loeb).
- 65) Cf. *Proclus’ Commentary on Plato’s Parmenides*, Translated by Glenn R. Morrow and John M. Dillon, Princeton University Press, 1987, pp. xxxviii–xliii.
- 66) Cf. H. Tarrant, ‘More on Zeno’s Forty Logoi,’ *ICS* 15, pp. 23–38.
- 67) Cf. Themistius, 199. 25.
- 68) Simplicius, 1013.31

ZENO'S FOUR PARADOXES AGAINST MOTION

- 69) Saying this I have in mind Zeno's two fragments B2 and B3 on which I shall give a detailed explanation later.
- 70) R. E. Allen considers that the abstract style of Zeno's argument does not fit the "vivid" imagery of the arguments preserved by Aristotle. See his *Plato's Parmenides*, p. 79.
- 71) In his treatise 'On What Is Not or Concerning Nature' Gorgias the sophist developed a discourse which may be regarded as a parody or a witty *reductio* of Eleatic metaphysics utilizing Eleatic logic at important points of which included some vivid images appealing to imagination: "a man flying," "chariots running over the sea," "Scylla," and "Chimaera." Cf. *DK*, B3, 79; 80.
- 72) Gorgias' fragment (79): 'οὐδὲ γὰρ ἂν φρονεῖ τις ἄνθρωπον ἰπτάμενον ἢ ἄρματα ἐν πελάγει τρέχοντα, εὐθέως ἄνθρωπος ἵπταται ἢ ἄρματα ἐν πελάγει τρέχει.' *DL* B3 (79).
- 73) 'Χελώνη καὶ λαγῶς περὶ ὀξύτητος ἤριζον. Καὶ δὴ προθεσμίαν στήσαντες καὶ τόπον ἀπηλλάγησαν. Ὁ μὲν οὖν λαγῶς διὰ τὴν φυσικὴν ὀκύτητα ἀμελήσας τοῦ δρόμου, πεσὼν παρ ὁδὸν ἐκοιμάτο. Ἡ δὲ χελώνη συνειδύα εαυτῇ βραδύτητα, οὐ διέλιπε τρέχουσα, καὶ οὕτω κοιμώμενον τὸν λαγῶν παραδραμοῦσα ἐπὶ τὸ βραβεῖον τῆς νίκης ἀφίκετο. Ὁ λόγος δηλοῖ ὅτι πολλακίς φύσιν ἀμελοῦσαν πόνος ἐνίκησεν.' (Émile Chambry, *Ésope, Fables*, Texte Établi et Traduit, Paris, Société d'édition Les Belles Lettres 1967) See also *Augustana Recension* 226; B. E. Perry, *Aesopica*, vol. I, Urbana, 1952; Cf. B. E. Perry (Edition & Translation), *Babrius and Phaedrus*, Harvard University Press (Loeb), pp. xi-xvi.
- 74) *Ibid.*
- 75) Cf. *Aesopica* 3; 26; 111; 177; 423; 428; 438; 447.
- 76) Cf. Plato, *Phaedo* 60d.
- 77) Cf. *DL*. II. 133–134. See also Gert-Jan van Dijk, *AINOI, AIOFOI, MYΘOI*, Fables in Archaic, Classical, and Hellenistic Greek Literature, With a Study of the Theory and Terminology of the Genre, Brill, 1997, pp. 344–345.
- 78) Cf. Hideya Yamakawa, 'The Mathematical Structure of Zeno's Paradoxes of Motion,' in Hideya Yamakawa, *Greek Philosophy ad the Moder World*, Studies in Greek Philosophy Series, International Center for Greek Phiosophy ad Culture,

- Ionia Publication, 1998, pp. 73-74.
- 79) For the detailed explanation of the matter, see my paper 'The Mathematical Structure of Zeno's Paradoxes of Motion,' in the above cited book, pp. 76-86.
- 80) See Cornford's note 1, p. 180 in Aristotle, *Physics* Books V-VIII (Loeb), translated by P. H. Wicksteed & F. M. Cornford. (First published 1934) and D. Ross, *Aristotle's Physics*, A Revised Text with Introduction and Commentary, Oxford, 1936, 1960, p. 658.
- 81) Cf. A27; H. Diels & W. Kranz, *Die Fragmente der Vorsokratiker*, Sechsten Auflage, 1964, S. 253.
- 82) Cf. Simplicius, 1015. 19 (Lee 30); Philoponus, 816. 30. (Lee 33).
- 83) Cf. Lee, *Op. cit.*, pp. 79-78.
- 84) Cf. W. D. Ross, *Op. cit.*, p. 658.
- 85) Cf. Parmenides, Fr. B8. 16; cf. also Hideya Yamakawa, 'Reductio ad Absurdum, On the Origin of Indirect Proof,' in Hideya Yamakawa, *Greek Philosophy and the Modern World*, Studies in Greek Philosophy Series No. 26, The International Center for Greek philosophy and Culture, 1998, pp. 59-62.
- 86) Cf. Lee, p. 64 ff. Epiphanius' reading is very similar to B4; therefore it may be regarded as a trusty copy of Zeno's original *Arrow*; On the other hand, in spite of a fact that Sextus Empiricus in *Pyrrh. Hyp.* III. 71 says that 'this argument is, in fact, that of Diodorus Cronos' (οὗτος δὲ ὁ λόγος ἔστι μὲν Διοδώρου τοῦ Κρόνου), because of its close resemblance to B4 and the *Arrow* I, we may take it as a copy of an argument of Diodorus Kronos, who in turn took it from Zeno's original argument.
- 87) τὸ κινούμενον οὐτε ἐν ᾧ ἐστὶ τόπω κινεῖται οὐτε ἐν ᾧ μὴ ἔστι.
- 88) τὸ κινούμενον ἦτοι ἐν ᾧ ἐστὶ τόπω κινεῖται ἢ ἐν ᾧ οὐκ ἔστι. καὶ οὐτε ἐν ᾧ ἐστὶ τόπω κινεῖται οὐτε ἐν ᾧ οὐκ ἔστιν· οὐκ ἄρα τι κινεῖται.
- 89) εἰ κινεῖται τι, ἦτοι ἐν ᾧ ἔστι τόπῳ κινεῖται ἢ ἐν ᾧ οὐκ ἔστιν. οὐτε δὲ ἐν ᾧ ἔστι μένει γὰρ ἐν αὐτῷ, εἴπερ ἐν αὐτῷ ἔστιν· ὅπου γὰρ τι μὴ ἔστιν, ἐκεῖ οὐδὲ δρᾶσαι τι οὐδὲ παθεῖν δύναται. οὐκ ἄρα κινεῖται τι.
- 90) τὸ κινούμενον ἦτοι ἐν ᾧ ἔστι τόπω κινεῖται ἢ ἐν ᾧ οὐκ ἔστι.
- 91) εἰ κινεῖται τι, ἦτοι ἐν ᾧ ἔστι τόπῳ κινεῖται ἢ ἐν ᾧ οὐκ ἔστιν. οὐτε δὲ ἐν ᾧ ἔστι.

ZENO'S FOUR PARADOXES AGAINST MOTION

- 92) Cf. Parmenides Fr. 8, 1-4, especially the following expression: 'μόνος δ' ἔτι μῦθος ὁδοῖο λείπεται ὡς ἔστιν.' (There still remains just one account of a way, that is.)
- 93) Cf. Aristotle, *Phys.*, 233b35-234a2.
- 94) Cf. Aristotle, *Phys.*, 220a21.
- 95) Literally he says that Zeno's conclusion that 'the arrow in motion *is at rest*' 'follows from the assumption that time is composed of a number of [indivisible] now; for if this is not granted the conclusion cannot be inferred.'
- 96) *Phys.*, 232a23-25.
- 97) Translation by R. P. Hardie and R. K. Gaye in J. Barnes (ed.), *The Complete Works of Aristotle*, The revised Oxford translation, Princeton University Press, 1984.
- 98) Cf. *Phys.*, 233b29-31.
- 99) Cf. also *Phys.*, 264a3-4.
- 100) Contra F. D. Miller, Jr., 'Aristotle against the Atomists,' in N. Kretzmann, *Infinity and Continuity in Ancient and Medieval Thought*, Cornell University Press, 1982, p. 101. Miller there insists that: the term *atomos* should be taken as simply "indivisible" and as referring to the points under fire in VI 1. Aristotle elsewhere refers to an *atomon nun*, which is a pointlike instant that should not, I think, be identified with the *atomos chronos* of VIII 8. For instants, like points, cannot be successive, whereas atomic times can be.
- 101) *Op. cit.*, 317a1-2.
- 102) Cf. Aristotle, *De Generatione et Corruptione*, 316a15-16; 316b15-16.
- 103) Cf. Andrew Pyle, *Atomism and Its Critics, Problem Areas associated with the Development of the Atomic Theory of Matter from Democritus to Newton*, Thoemmes Press, 1995, p. 1.
- 104) See Grünbaum, 'Zeno's Metrical Paradox of Extension' in W. Salmon (ed.), *Zeno's Paradoxes*, Bobbs-Merrill, Indianapolis and New York, 1970.
- 105) Cf. Simplicius, 139.27; See Lee, p. 13.
- 106) Cf. DL, fr. 2; Simplicius, 139. 5.
- 107) Contra R. Sorabji, 'Atoms and Time Atoms,' in N. Kretzmann, *Infinity and*

Continuity in Ancient and Medieval Thought, Cornell University Press, 1982, pp. 59–62. Cf. Sextus, *Adv. Math.*, X 119–120. We should realize that the original idea of an atomic “present” or “now” is never remote from our everyday time-experience.

- 108) We should remember that Hesiod in the *Works and Days* often refers to the time limit where one ought to work hard. Impressing upon Perseus the importance of labor he preaches as follows: ‘When Zeus has finished sixty wintry days after the solstice, then the star Arcturus leaves the holy stream of Ocean and first rises brilliant at dusk. After him the shrilly wailing daughter of Pandion, the swallow, appears to men when spring is just beginning. Before she comes, prune the vines, for it is best so (Huh G. Evelyn-White, *Hesiod, Works and Days*, Loeb Classical Library 57, 1936, 1995, pp. 45–46). Reading this passage we can easily visualize a figure of plowman who encourages himself and says: ‘*Now it is time* to prune the vines.’ For a plowman the interval between the rising of Arcturus and the first appearance of the swallow is nothing but a “now.” (Cf. *Hesiod, Works and Days*, 176: ‘νῦν γὰρ δὴ ...’) Such a “now” is not indivisible. It is but a time span which may be correlated to a number of perceptible events. However we can imagine a situation where such an interval becomes increasingly narrower and arrives eventually at a threshold wherein the difference between events can no longer be discriminated. And such a perceptual threshold affords us the notion of moment as a minimal and indivisible time which Jakob von Uexküll in *Streifzüge durch die Umwelten von Tieren und Menschen* called “das kleinste unteilbare Zeitgefäß”: ‘Die Momente sind die kleinsten unteilbaren Zeitgefassen, weil sie der Ausdruck von unteilbaren Elementarempfindungen, den sogenannten Momentzeichen, sind. Für den Menschen beträgt, wie bereits gesagt, die Dauer eines Momentes 1/18 Sekunde. Und zwar ist der Moment für alle Sinnesgebiete der gleiche, weil alle Sinnesempfindungen von dem gleichen Momentzeichen begleitet werden. Achtzehn Luftschwingungen werden nicht mehr unterscheiden, sondern als ein einheitlicher Ton gehört. Es hat sich gezeigt, daß der Mensch 18 Stöße, die seine Haut treffen, als gleichmäßigen Druck empfindet. Die Kinematographie bietet uns die Möglichkeit, Bewegungen

ZENO'S FOUR PARADOXES AGAINST MOTION

der Außenwelt in dem uns gewohnten Tempo auf die Leinwand zu werfen. Dabei folgen sich die einzelnen Bilder in kleinen Rucken von 1/18 Sekunden.' (Jakob von Uexküll und Georg Kriszat, *Streifzüge* durch die Umwelten von Tieren und Menschen, S. Fischer Verlag, 1970, p. 33). It is but a step from this notion of the perceptual "now" to one of the atomic "now," whereas there is a great disparity between this and the Aristotelian indivisible "now." The Aristotelian definition of "now" is so modern and sophisticated that it is reminiscent of Dedekind's definition of the real number. (J. W. R. Dedekind, *Stetigkeit und irrationale Zahlen*, 1872; 2 Aufl. 1965; *Mathematische Werke* Bd. 2) It is very unlikely that Zeno attributed such a refined theory to his opponents.

- 109) Barnes' words; according to Barnes an "architectonic" structure of Zeno's four paradoxes is a mere product of scholars' "neat fantasy." Jonathan Barnes, *The Presocratic Philosophers*, Routledge & Kegan Paul, The revised edition in one volume, 1986, p. 285.
- 110) Cf. *Op. cit.*, p. 291.
- 111) Cf. Paul Feyerabend, *Against Method*, Revised Edition, Verso, 1988. Literally he says that 'the consistency condition which demands that new hypotheses agree with accepted theories is unreasonable because it preserves the older theory, and not the better theory. Hypotheses contradicting well-confirmed theories give us evidence that cannot be obtained in any other way.' p. 5.
- 112) Patricia Curd, *The Legacy of Parmenides*, Eleatic Monism and Later Presocratic Thought, Princeton University Press, 1998, p. 179.
- 113) Cf. Aristotle, *Physics*, 231b18-20.
- 114) *Op. cit.*, 219a12-13.
- 115) Cf. Aristotle, *Physics*, Books V-VIII, Loeb Classical Library, p. 180.
- 116) Cf. Bertrand Russell, *Our Knowledge of the External World*, George Allen & Unwin Ltd., 1938, p. 179.
- 117) This principle underlies the 'ἤρεμεϊ πᾶν ἢ κινεῖται' as I have insisted.
- 118) Cf. the above quoted *Phys.*, Z9, 239: 'ὅταν ᾗ κατὰ τὸ ἴσον,' κτλ.
- 119) Cf. the above Sextus Empiricus: 'μένει γὰρ ἐν αὐτῷ.' I think, this phrase reflects Zenonian genuine expression; cf. Parmenides B8. 26-30: 'But change-

less within the limits of great bonds it *exists* without beginning or ceasing, ... *remaining* the same and in the same place it *lies* on its own (ταῦτόν τ' ἐν ταῦτόν τε μένον καθ' ἑαυτό) and thus fixed it will *remain*.'

120) This shows that it is not the case that Zeno did not argue the counterpart of motion (that is, rest)..

121) This negates the possibility of the above mentioned (4) 'The same arrow is and is not in motion at the same time.' Hegelians or Marxists may appeal to this possibility; but Zeno rejects it. Zeno succeeded to Parmenidean law of the excluded middle: 'ἢ δὲ κρίσις περὶ τούτων ἐν τῷδ' ἔστιν· ἔστιν ἢ οὐκ ἔστιν.' (B8. 15-16).

122) Parmenidean disjunction is a strong one; A or B , and not the case that A and B at the same time; in modern notation: $(A \vee B) \wedge \neg (A \wedge B)$. A is contradictory to B , i.e., *tertium non datur*. Thus, it is the case that ' $(A \rightarrow \neg B) \wedge (\neg B \rightarrow A)$ ' and so that $A \equiv \neg B$; that is, the opposition between A and B is nothing but one between A and $\neg A$. Indeed, in fragment 8. 16 the goddess declares: ἔστιν ἢ οὐκ ἔστιν (is or is not); so the major premise of the disjunctive syllogism of Parmenides originally the following: $A \vee \neg A$; and the minor premise is given by a *reduction ad absurdum* argument: ' $\neg A$ leads to an absurdity (or $\neg A$ results in an impossibility), therefore not $\neg A$.' Then, given the major premise and the minor premise, A is concluded as follows: 1. $A \vee \neg A$, 2. ($\neg A \rightarrow$ Contradiction) $\rightarrow \neg \neg A$, 3. $\neg \neg A$, therefore A . For the detail of Parmenidean reduction argument see my paper '*Reductio ad absurdum, on the Origin of Indirect Proof,*' in Hideya Yamakawa, *Greek philosophy and the Modern World*, International Center for Greek Philosophy and Culture, Ionia Publication, 1998, pp. 53-72.

123) Cf. W. D. Ross, *Op. cit.*, p. 74.

124) Plato, *Parmenides*, 166c: 'If Unity is not, the others will not be either one or many. Neither will they appear one or many, for Unity, if it does not exist, cannot seem to be present to anything or be conceived by anything. And since the others cannot appear or be conceived to be one, they cannot appear or be conceived to be one, they cannot appear or be conceived to be many. It follows that they can have no other characters. In sum: if Unity is not, nothing is.' (Translation by R.

ZENO'S FOUR PARADOXES AGAINST MOTION

- E. Allen; See R. E. Allen, *Plato's Parmenides*, Translated with Comment, Revised edition, Yale University Press, 1997, p. 338.)
- 125) Cf. Liddell and Scott, *Greek-English Lexicon*, 9th edition, Oxford, 1940, (Reprinted 1966)
- 126) Cf. Cornford insisted that the target of Zeno's attack was Pythagorean "number-atomism." See F. M. Cornford, *Plato and Parmenides*, Kegan Paul, 1939, Chap. 3 'Zeno and Pythagorean atomism.'
- 127) Cf. R. K. Gaye, 'On Aristotle, *Phys. Z*, IX, 239b33-240a18,' *J. Phil.* (1908), pp. 95-116.
- 128) But, if the fourth argument is Zeno's genuine one, it is unlikely that only this is irrelevant to the race, because all of the other paradoxes, the *Dichotomy*, the *Achilles and Tortoise*, and the *Arrow*, can be considered to have their relevance to some kind of races in the stadium; that is to say, the *Dichotomy* and the *Achilles* to the footrace and the *Arrow* to the archery contest.
- 129) Cf. W. K. C. Guthrie, *A History of Greek Philosophy*, Vol. II, Cambridge University Press, 1969, p. 96.
- 130) Cf. *Hsi Yu Chi*.
- 131) Cf. Lee, *op. cit.*, p. 59.
- 132) Cf. Lee, *op. cit.*, p. 98.
- 133) Cf. fr., 8. 6.
- 134) Cf. fr., 8. 22.
- 135) Cf. fr., 8. 26.
- 136) Cf. Plato, *Parmenides*, 128b-e.
- 137) See Rafael Ferber, *Zenon's Paradoxien der Bewegung und die Struktur von Raum und Zeit*, 2., durch gesehene und um ein Nachwort erweiterte Auflage, Franz Steiner Verlag Stuttgart, 1995, S. 119. It is very significant that R. Ferber too paid attention to Parmenides' "σφαίσης ... ὄγκω." However, his reference to ὄγκω is self-contradictory in the framework of his own interpretation of the *Stadium*. Witty though his idea is, his hypothesis on "ὄγκοι" cannot get along with his interpretation of the *Stadium*. Ferber regards each of the ὄγκοι as a spherical object like a golf ball or a foot ball. Of course, It is not so unsuitable

idea to imagine the ὄγκος as something like ball; see Homeros, *Od.*, 6. 100. Then, is it divisible or indivisible? If it were divisible, it could not have any intrinsic relevance to Parmenides' "what is," because "τὸ ἕν" is *par excellence* "indivisible" (οὐδὲ διαίρετόν). But if it were indivisible, then its atomic property will damage his interpretation of the *Stadium*. In reality he rejects explicitly the atomic interpretation of the *Stadium*. Criticizing Mansfeld's view of the *Stadium* he says as follows: 'Mansfeld's Interpretation besteht nun darin, dass die Massen der beiden Reihen diskrete Grössen sind. ... Beim Stadium läuft dieses Argument aber auf neue Formulierung der Relativität der Bewegung hinaus, kombiniert mit der Hypothese, dass die Massen Atome sind. Aber die Hypothese der Unteilbarkeit der Massen ist schon lange widerlegt, und die Relativität der Bewegung schon im Argument vom Pfeil akzeptiert worden.' See, Rafael Ferber, *Op. cit.*, pp. 115-116. To tell the truth, the ὄγκοι in plural have no significant role within the framework of Ferber's interpretation of the *Stadium*. They are irrelevant to the essential argument of the *Stadium*. In addition, it matters little whether they move towards the opposite direction or not. In any way, they are, according to Ferber, products fabricated by Aristotle.

138) Parmenides, Fr. 8. 25.

139) Democritus wrote a work titled with 'On a difference of opinion, or on the contact of a circle and a sphere which was, according to Heath in *A History of Greek Mathematics*, Vol. I *From Thales to Euclid*, Oxford, 1921 (First edit.), 1965, pp. 178-179, in which he retorted Protagoras who attacked geometry in his book *περὶ τῶν μαθημάτων* (*On mathematics*) saying that: 'A material circle does not in actual fact touch a ruler at one point only.' The fact obviously shows that among Protagoras' contemporary mathematicians the topic of the contact of a circle or sphere with its tangent was a well-known problem. Plato in the *Seventh Letter* spoke of the circles that are drawn I geometric exercises or are turned on the lathe and declared that each one of them is everywhere in contact with the straight, in order to defend his own theory of Forms (*See Seventh Letter*, 343a); cf. A. Wedberg, *Plato's Philosophy of Mathematics*, Almqvist & Wiksell, Stockholm, 1955, p. 57; cf. also Aristotle, *Metaphysics*, B. 2. 998a3; cf. also W. D.

ZENO'S FOUR PARADOXES AGAINST MOTION

Ross, *Aristotle's Metaphysics*, A Revised Text with Introduction and Commentary, Vol. I, Oxford, 1924 (First edit.), 1958, p. 232. By the way, it is an interesting fact that we read a Zenonian paradox concerning the 'Millet Seed' in Simplicius' comment to Aristotle, *Phys.* H5. 250a19 (1108. 18) which preserves a dialogue between Zeno and Protagoras which runs as follows: 'By this means he solves the conundrum which Zeno the Eleatic asked Protagoras the sophist. "Tell me, Protagoras," he said, "does a single grain of millet or the ten thousandth part of a grain make any sound when it falls?" And when Protagoras said it did not, "Then", asked Zeno, "does a bushel of millet make any sound when it falls or not?" Protagoras answered that it did, whereupon Zeno replied, "But surely there is some ratio between a bushel of millet and a single grain or even the ten thousandth part of a grain"; and when this was admitted, "But then surely", Zeno said, "the ratios of the corresponding sounds to each other will be the same: for as the bodies which make the sounds are to one another, so will the sounds be to one another. And if this is so, and if the bushel of millet makes a sound, then the single grain of millet and the ten thousandth part of a grain will make a sound." This was the way Zeno used to put his questions.' (translation by Lee) In spite of Diogenes Laertius (III. 48) some said that Zeno was the first to write dialogues, it is unlikely that he was an author of this dialogue in which he himself the leading character. Therefore it is provably sure that the above quoted dialogue was written by others who had some interests with the relationship between Zeno and Protagoras. Protagoras (born in Abdera not later than 490 B. C. and probably died soon after 421 B. C.) and Zeno (born about 490 B. C.) were contemporaries so that it was quite possible that they had met. And this means that Zeno had through knowledge of the problem of the contact of a circle or sphere with its tangent.

- 140) See Parmenides, frag. 8, 22-25: 'Nor is it divided, since it all exists alike; nor is it more here and less there, which would prevent it from holding together, but it is all full of being. So it is all continuous: for what is draws near to what is (ἐὸν γὰρ ἔόντι περάζει).' The last word 'ἐὸν γὰρ ἔόντι περάζει' should be interpreted as an expression of the continuity of the what is.

- 141) Cf. Lee, *Op. cit.*, p. 86.
- 142) Cf. Lee, *Op. cit.*, p. 86.
- 143) Cf. Lee, *Op. cit.*, p. 87.
- 144) Cf. Rafael Ferber, *Zenons Paradoxien der Bewegung und die Struktur von Raum und Zeit*, 2., durchgesehene und um ein Nachwort erweiterte Auflage, Franz Steiner Verlag Stuttgart, 1995, pp. 23-26; pp. 114-118.
- 145) Cf. Rafael Ferber, *Op. cit.*, p. 30.
- 146) Cf. Rafael Ferber, *Op. cit.*, p. 3.
- 147) Cf. Rafael Ferber, *Op. cit.*, p. 24.
- 148) Cf. Ross, *Op. cit.*, pp. 663-664.
- 149) Cf. XXIII, 495-498.
- 150) Cf. XXIII, 375-382.
- 151) Cf. XXIII, 417-441.
- 152) Cf. Lee, *Op. cit.*, pp. 87-88.

SUMMARY OF 'ZENO'S FOUR PARADOXES AGAINST MOTION'

Hideya YAMAKAWA

Notifying my intention in the study of Zeno's four paradoxes to be continued in a long series, I would like to convey my true motive. I have no interest in dealing with Zeno's discourse qua "puzzle." And much less I intend to issue some new and unexplored "solutions" of Zeno's discourses qua "puzzles." which might be appreciated only by a small circle of specialists.

My original intention is to show a fact that something "invisible" lurks behind the "visible" surface of Zeno's discourses. Zeno's "paradoxes" as "visible" strata are pregnant of the "invisible" meaning. However, the "invisible" meaning in question is not separated from its "visible" layer. The "visible" thing itself assumes the meaning of the "invisible." Thus, the "invisible" is a secret counterpart of the "visible."

The subject-matter Zeno called into question and indeed Zeno himself hide behind the "visible" surface qua "puzzle." Therefore, in order to excavate Zeno's "invisible" figure and bring it to light successfully, we must remove a large quantity of surface soil which has covered Zeno's true identity. Thus breaking through the bulky layers of "paradoxes," we have, by all means, to get to the solid rock of Zeno's thought.

Thus, in this paper 'Zeno's Four Paradoxes against Motion' I shall begin to show my provisional outlook of Zeno's paradoxes and then proceed to digging up the first layer of Zeno's paradoxes, which a large quantity of surface soil qua

“puzzles” has covered. Thus I hope that I could show you some invisible and hidden dimensions of Zeno’s paradoxes, which had gone unnoticed for a long time.